Sequence and Series

EXERCISES

ELEMENTRY

Q.1 (2)

We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$

$$=1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+........$$

$$=\sqrt{2}[1+2+3+4+..........\text{upto 24 terms}]$$

$$=\sqrt{2}\times\frac{24\times 25}{2}=300\sqrt{2}$$

Q.2 (3)

Suppose that first term and common difference of A.P.'s are A and D respectively.

Now, p^{th} term $=A+(p-1)D=a$ …..(i)

q^{th} term $=A+(q-1)D=b$ ......(ii)

and r^{th} term $=A+(r-1)D=c$ …..(iii)

So, $a(q-r)+b(r-p)+c(p-q)$

$$=a\left\{\frac{b-c}{D}\right\}+b\left\{\frac{c-a}{D}\right\}+c\left\{\frac{a-b}{D}\right\}$$

$$=\frac{1}{D}(ab-ac+bc-ab+ca-bc)=0\ .$$

Q.3 (3)

$$T_m=a+(m-1)d=\frac{1}{n}$$

and $T_n=a+(n-1)d=\dfrac{1}{m}$

On solving $a=\dfrac{1}{mn}$ and $d=\dfrac{1}{mn}$

$$\therefore\ T_{mn}=a+(mn-1)d=\frac{1}{mn}+(mn-1)\frac{1}{mn}=1$$

Q.4 (2)

The given number are in A.P.

$$\therefore 2\log_9(3^{1-x}+2)=\log_3(4.3^x-1)+1$$

$$\Rightarrow\ 2\log_{3^2}(3^{1-x}+2)=\log_3(4.3^x-1)+\log_3 3$$

$$\Rightarrow\ \frac{2}{2}\log_3(3^{1-x}+2)=\log_3[3(4.3^x-1)]$$

$$\Rightarrow\ 3^{1-x}+2=3(4.3^x-1)$$

$$\Rightarrow\ \frac{3}{y}+2=12y-3,\ \text{ where } y=3^x$$

$$\Rightarrow\ 12y^2-5y-3=0$$

$$y=\frac{-1}{3}\text{ or }\frac{3}{4}\Rightarrow 3^x=\frac{-1}{3}\ \text{ or }\ 3^x=\frac{3}{4}$$

$$x=\log_3(3/4)\ \Rightarrow x=1-\log_3 4\ .$$

Q.5 (3)

Let S_n and S'_n be the sums of n terms of two A.P.'s and T_{11} and T'_{11} be the respective 11^{th} terms, then

$$\frac{S_n}{S'_n}=\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}=\frac{7n+1}{4n+27}\ \Rightarrow$$

$$\frac{a+\frac{(n-1)}{2}d}{a'+\frac{(n-1)}{2}d'}=\frac{7n+1}{4n+27}$$

Now put $n=21$,

we get $\dfrac{a+10d}{a'+10d'}=\dfrac{T_{11}}{T'_{11}}=\dfrac{148}{111}=\dfrac{4}{3}$.

Note : If ratio of sum of n terms of two A.P.'s are given in terms of and ratio of their terms are to be found then put . Here we put .

Q.6 (3)

Let $S_{Even}=2+4+6+8+........\infty$ …..(i)

and $S_{Odd}=1+3+5+7+9+.......\infty$ …..(ii)

Sum $S_E\dfrac{n}{2}\left[4+(n-1)2\right]=\dfrac{n}{2}[2n+2]=\dfrac{n}{2}2(n+1)$

and $S_O=\dfrac{n}{2}[2+(n-1)2]=\dfrac{n}{2}(2n)$

Now $\dfrac{S_E}{S_O}=\dfrac{(n+1)}{n}$ or $S_E:S_O=(n+1):n$

Q.7 (3)

$$S_{2n}-S_n=\frac{2n}{2}\{2a+(2n-1)d\}-\frac{n}{2}\{2a+(n-1)d\}$$

$$= \frac{n}{2}\{4a + 4nd - 2d - 2a - nd + d\} = \frac{n}{2}\{2a + (3n-1)d\}$$

$$= \frac{1}{3}.\frac{3n}{2}\{2a + (3n-1)d\} = \frac{1}{3}S_{3n}$$

Q.8 (4)

Let $S = 1 + 2 + 3 + ... + 100$

$$= \frac{100}{2}(1 + 100) = 50(101) = 5050$$

Let $S_1 = 3 + 6 + 9 + 12 + ... + 99$

$$= 3(1 + 2 + 3 + 4 + ... + 33)$$

$$= 3.\frac{33}{2}(1 + 33) = 99 \times 17 = 1683$$

Let $S_2 = 5 + 10 + 15 + ... + 100$

$$= 5(1 + 2 + 3 + ... + 20)$$

$$= 5.\frac{20}{2}(1 + 20) = 50 \times 21 = 1050$$

Let $S_3 = 15 + 30 + 45 + ... + 90$

$$= 15(1 + 2 + 3 + ... + 6)$$

$$= 15.\frac{6}{2}(1 + 6) = 45 \times 7 = 315$$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$$= 5050 - 1683 - 1050 + 315 = 2632.$$

Q.9 (2)

$S_{2n} = 3S_n$

$$\Rightarrow \frac{2n}{2}\{2a + (2n-1)d\} = 3.\frac{n}{2}\{2a + (n-1)d\}$$

$2a = (n+1)d$

Put $2a = (n+1)d$ in $\frac{S_{3n}}{S_n}$, we get its value 6.

Q.10 (4)

Suppose that required numbers a and b. Therefore according to the conditions $a = \frac{1}{b}$

and $\frac{a+b}{2} = \frac{13}{12} \Rightarrow a + b = \frac{13}{6}$

$$\Rightarrow a + \frac{1}{a} = \frac{13}{6} \Rightarrow 6a^2 - 13a + 6 = 0$$

$$\Rightarrow \left(a - \frac{3}{2}\right)\left(a - \frac{2}{3}\right) = 0 \Rightarrow a = \frac{3}{2} \text{ and } b = \frac{2}{3}$$

$$\Rightarrow a = \frac{2}{3} \text{ and } b = \frac{3}{2}$$

Trick : Find the A.M. of option (1), (2), (3), (4) one by one.

Q.11 (4)

Given that x, 2x + 2, 3x + 3 are in G.P.

Therefore, $(2x + 2)^2 = x(2x + 2)^2 = x(3x + 3)$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x + 4)(x + 1) = 0 \Rightarrow x = -1, -4$

Now first term a = x

Second term $ar = 2(x+1) \Rightarrow r = \frac{2(x+1)}{x}$

then 4th term $= ar^3 = x\left[\frac{2(x+1)}{x}\right]^3 = \frac{8}{x^2}(x+1)^3$

Putting x = –4

We get $T_4 = \frac{8}{16}(-3)^3 = -\frac{27}{2} = -13.5$

Q.12 (2)

$T_6 = 32$ and $T_8 = 128 \Rightarrow ar^5 = 32$ .....(i)

and $ar^7 = 128$ .....(ii)

Dividing (ii) by (i), $r^2 = 4 \Rightarrow r = 2$

Q.13 (1)

Series is a G.P. with $a = 0.9 = \frac{9}{10}$ and

$r = \frac{1}{10} = 0.1$

$$\therefore S_{100} = a\left(\frac{1 - r^{100}}{1 - r}\right) = \frac{9}{10}\left(\frac{1 - \frac{1}{10^{100}}}{1 - \frac{1}{10}}\right) = 1 - \frac{1}{10^{100}}.$$

Q.14 (2)

Given series $6 + 66 + 666 + ... +$ upto n terms

$$= \frac{6}{9}(9 + 99 + 999 + ... \text{upto terms})$$

$$= \frac{2}{3}(10 + 10^2 + 10^3 + ... + \text{ upto terms})$$

$$= \frac{2}{3}\left(\frac{10(10^n - 1)}{10 - 1} - n\right) = \frac{1}{27}[20(10^n - 1) - 18n]$$

$$= \frac{2(10^{n+1} - 9n - 10)}{27}$$

Q.15 (1)

$\because$ a, b, c are in G.P.

$$\therefore \frac{b}{a}=\frac{c}{b}=r$$

$$\Rightarrow \frac{b^2}{a^2}=\frac{c^2}{b^2}=r^2 \; a^2,\; b^2, c^2 \text{ are in G.P.}$$

Q.16 (4)

Here $\frac{a}{1-r}=4$ and $ar=\frac{3}{4}$. Dividing these,

$r(1-r)=\frac{3}{16}$ or $16r^2-16r+3=0$

or $(4r-3)(4r-1)=0$

$r=\frac{1}{4},\frac{3}{4}$ and $a=3,1$ so $(a,r)=\left(3,\frac{1}{4}\right),\left(1,\frac{3}{4}\right)$.

Q.17 (2)

As given $G=\sqrt{xy}$

$$\therefore \frac{1}{G^2-x^2}+\frac{1}{G^2-y^2}=\frac{1}{xy-x^2}+\frac{1}{xy-y^2}$$

$$=\frac{1}{x-y}\left\{-\frac{1}{x}+\frac{1}{y}\right\}=\frac{1}{xy}=\frac{1}{G^2}.$$

Q.18 (4)

It is an arithmetico-geometric series

$$S_\infty=\frac{a}{1-r}+\frac{dr}{(1-r)^2}=\frac{1}{1-\frac{1}{2}}+\frac{2}{\left(1-\frac{1}{2}\right)^2}=\frac{2}{\frac{1}{2}}+\frac{2}{\frac{1}{4}}$$

$$=4+8=12$$

Q.19 (3)

The sum of the first n terms is

$$S_n=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{2^3}\right)+\left(1-\frac{1}{2^4}\right)$$

$$+......+\left(1-\frac{1}{2^n}\right)=n-\left\{\frac{1}{2}+\frac{1}{2^2}+.....+\frac{1}{2^n}\right\}$$

$$=n-\frac{1}{2}\left(\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right)=n-\left(1-\frac{1}{2^n}\right)=n-1+2^{-n}.$$

Trick : Check for $n=1, 2$ i.e. $S_1=\frac{1}{2}, S_2=\frac{5}{4}$ and

(3) $\Rightarrow S_1=\frac{1}{2}$ and $S_2=2+2^{-2}-1=\frac{5}{4}$.

Q.20 (4)

Suppose that x to be added then numbers 13, 15, 19 so that new numbers $x+13, 15+x, 19+x$ will be in H.P.

$$\Rightarrow (15+x)=\frac{2(x+13)(19+x)}{x+13+x+19}$$

$$\Rightarrow x^2+31x+240=x^2+32x+247 \Rightarrow x=-7$$

Trick : Such type of questions should be checked with the options.

Q.21 (1)

Here 5^{th} term of the corresponding

A.P. $=a+4d=45$ .....(i)

and 11^{th} term of the corresponding

A.P. $=a+10d=69$ .....(ii)

From (i) and (ii), we get $a=29, d=4$

Therefore 16th term of the corresponding A.P. $=a+15d=29+15\times 4=89$.

Hence 16th term of the H.P. is $\frac{1}{89}$.

Q.22 (4)

Considering corresponding A.P.

$a+6d=10$ and $a+11d=25 \Rightarrow d=3, a=-8$

Hence term of the corresponding H.P. is $\frac{1}{49}$

Q.23 (1)

As given $H=\frac{2pq}{p+q}$

$$\therefore \frac{H}{p}+\frac{H}{q}=\frac{2q}{p+q}+\frac{2p}{p+q}=\frac{2(p+q)}{p+q}=2.$$

Q.24 (3)

A.M. $=\frac{a+b}{2}=A$ and G.M. $=\sqrt{ab}=G$

On solving a and b are given by the values

$A \pm \sqrt{(A+G)(A-G)}$.

Trick : Let the numbers be 1, 9. Then A=5 and G=3. Now put these values in options.

Here (3) $\Rightarrow 5 \pm \sqrt{8 \times 2}$ i.e. 9 and 1.

Q.25 (4)

Let $A = \frac{a+b}{2}, G = \sqrt{ab}$ and $H = \frac{2ab}{a+b}$.

Then $G^2 = ab$ .....(i)

and $AH = \left(\frac{a+b}{2}\right) . \frac{2ab}{a+b} = ab$ .....(ii)

From (i) and (ii), we have $G^2 = AH$

Q.26 (1)

Since a^2, b^2, c^2 be in A.P. Then

$$b^2 - a^2 = c^2 - b^2$$

$$\Rightarrow (b-a)(b+a) = (c-b)(c+b) \Rightarrow \frac{b-a}{c+b} = \frac{c-b}{b+a}$$

$$\Rightarrow \frac{(b-a)(a+b+c)}{(c+a)(b+c)} = \frac{(c-b)(a+b+c)}{(a+b)(c+a)}$$

$$\Rightarrow \frac{b^2+bc-ac-a^2}{(c+a)(b+c)} = \frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$$

$$\Rightarrow \frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$$

Hence $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ be in A.P.

Q.27 (4)

Here $T_n = \frac{n(n+1)}{2}$

Therefore $S_n = \frac{1}{2}\left\{\Sigma n^2 + \Sigma n\right\} = \frac{n(n+1)(n+2)}{6}$

Q.28 (1)

$$\frac{1^3+2^3+3^3+4^3+.........+12^3}{1^2+2^2+3^2+4^2+........+12^2}$$

$$= \frac{\left(\sum_{n=1}^{12} n^3\right)}{\left(\sum_{n=1}^{12} n^2\right)} = \left[\frac{n(n+1)}{2}\right]^2 \times \frac{6}{n\,(n+1)\,(2n+1)}$$

$$= \frac{3}{2} . \frac{n\,(n+1)}{(2n+1)} = \frac{3}{2} . \frac{12\,.\,13}{25} = \frac{234}{25}$$

[Putting n=12].

Q.29 (2)

Here, $T_n = 3 + n(n-1) = 3 + n^2 - n$

Now sum $S = \Sigma T_n = \Sigma(3 + n^2 - n)$

$$= 3n + \frac{1}{6}n(n+1)(2n+1) - \frac{n(n+1)}{2}$$

$$= \frac{1}{6}n(n+1)[2n+1-3] + 3n = \frac{n^3+8n}{3}$$

Q.30 (1)

$$T_n = \frac{1+2+3+.....+n}{n} = \frac{n(n+1)}{2n} = \frac{1}{2}(n+1)$$

Hence, $S = \frac{1}{2}(\Sigma n + n) = \frac{1}{2}\left\{\frac{n(n+1)}{2} + n\right\} = \frac{n(n+3)}{4}$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (4)

$S = \frac{2p+1}{2}$ $[2(p^2 + 1) + 2p]$

$= (2p + 1)\ (p^2 + 1 + p)$

$= 2p^3 + 3p^2 + 3p + 1 = p^3 + (p + 1)^3$

Q.2 (4)

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow 3\ (a_1 + a_{24}) = 225$

(sum of terms equidistant from beginning and end are equal)

$a_1 + a_{24} = 75$

Now $a_1 + a_2 + ... + a_{23} + a_{24} = \frac{24}{2}\ [a_1 + a_{24}] = 12 \times 75 = 900$

Q.3 (3)

Sum of the integer divided by 2

= 2 + 4 + ...+ 98 + 100

$= \frac{50}{2}\ [2.2 + (50 - 1)2]$

= 50 [51] = 2550

Sum of the integer divided by 5

= 5 + 10 +..................95 + 100

$= \frac{20}{2}\ [5 + 100] = 1050$

Sum of the integer divided by 10

$\frac{10}{2}$ [10 + 100] = 550

Sum of the integers divided by 5 or 10 = 2550 + 1050 – 550 = 3050

Q.4 (2)

Let a, a + d, a + 2d .... are Interior angles

$\because$ sum of interior angles = (n – 2) π , where n is the number of sides

$\therefore$ a = 120°, d = 5°

$\Rightarrow \frac{n}{2}[240° + (n – 1) 5°] = (n – 2) 180°$

$\Rightarrow n^2 = 25n – 144 \Rightarrow$ n = 16, 9 but n ≠ 16

because if n = 16, then an interior angle will be 180° which is not possible So n = 9

Q.5 (1)

$$\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}[2a+(kx-1)d]}{\frac{x}{2}[2a+(x-1)d]} = k\left[\frac{2a+(kx-1)d}{2a+(x-1)d}\right]$$

If 2a – d = 0,then $\frac{S_{kx}}{S_x}$ is independent of x

So d = 2a

Q.6 (4)

x∈R

$5^{1+x} + 5^{1-x}$, a/2 , 5^{2x} $+5^{-2x}$ are in A.P

$a = (5^{2x} + 5^{-2x}) + (5^{1+x} + 5^{1-x})$

$a = (5^{2x} + 5^{-2x}) + 5(5^{x} + 5^{-x})$

$= (5^{x} - 5^{-x})^2 + 2 + 5\,(5^{x/2} - 5^{-x/2})^2 + 10$

$a = 12 + (5^{x} - 5^{-x})^2 + 5(5^{x/2} - 5^{-x/2})^2$

$\Rightarrow a \geq 12$

Q.7 (4)

$$S = \frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + ...+ \frac{1}{\log_{2^n} 4}$$

$$= \frac{1}{2} + \frac{1}{1} + \frac{1}{2/3} + ...+ \frac{1}{2/n}$$

$$= \frac{1}{2} + 1 + \frac{3}{2} + \frac{4}{2} + ...+ \frac{n}{2}$$

$$= \frac{n(n+1)}{4}$$ **Ans**

Q.8 (1)

Given that

$S = 1^2 - 2^2 + 3^2 - 4^2 + ... 2002^2 + 2003^2$

$= 1 + (3^2 - 2^2) + (5^2 - 4^2) + ... + (2003^2 - 2002^2)$

= 1 + 2 + 3 + 4 + 5 + ... + 2002 + 2003

$= \frac{2003}{2}$ [1 + 2003] = 2003 (1002) = (2000 + 3) (1000 + 2) = 2007006

Q.9 (2)

Let AP be A, A + d, A + 2d, ...

$T_p = a, T_q = b$

$\therefore$ a = A + (p – 1) d ...(i)

b = A + (q –1) d ...(ii)

$\Rightarrow$ subtract (i) & (ii) $\frac{a-b}{p-q} = d$

add (i) & (ii) a + b = 2A + (p + q – 1) d – d

$\Rightarrow$ 2A + (p + q – 1) d = (a + b) + d

$$S_{p+q} = \frac{(p+q)}{2}[2A + (p + q – 1) d]$$

$$= \frac{(p+q)}{2}\left[a+b+\frac{a-b}{p-q}\right]$$

Q.10 (2)

$$\frac{(54-3)}{n+1} = d$$

$$d = \frac{51}{n+1}$$

$$\frac{A_8}{A_{n-2}} = \frac{3}{5}$$

$$\Rightarrow \frac{3+8\frac{51}{n+1}}{3+(n-2)\frac{51}{n+1}} = \frac{3}{5}$$

$$\Rightarrow \frac{3n+3+408}{3n+3+51n-102} = \frac{3}{5}$$

$\Rightarrow$ 15n + 2055 = 162n – 297

$\Rightarrow$ 147 n = 2352

n = 16

Q.11 (2)

Let the GP be a, ar^2, ar^3, ...

Now, $ar^2 = 4$

$\therefore a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4 = a^5 r^{10} = (ar^2)^5 = 4^5$

Q.12 (2)

Let the GP be a, ar^2, ar^3, ...

We know that sum of G.P. is possible $\Rightarrow$ | r | < 1

$$S = \frac{a}{1-r} \Rightarrow r = \left(1-\frac{a}{S}\right)$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a\left(1-\left(1-\frac{a}{S}\right)^n\right)}{\frac{a}{S}} = S\left[1-\left(1-\frac{a}{S}\right)^n\right]$$

Q.13 (2)

Given,

$a_1 = 2$, & $\frac{a_{n+1}}{a_n} = \frac{1}{3} = r$,

$$\sum_{r=1}^{20} a_r = \frac{a_1(1-r^{20})}{1-r} = \frac{2\left(1-\left(\frac{1}{3}\right)^{20}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^{20}}\right)$$

Q.14 (3)

Given that,

$x^2 - 3x + a = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$ $\qquad$ $x^2 - 12x + b = 0 \begin{matrix} \gamma \\ \delta \end{matrix}$

Also, $\alpha, \beta, \gamma, \delta$ in increasing G.P.

Let $\alpha, \beta, \gamma, \delta$ be $\alpha, \alpha r, \alpha r^2, \alpha r^3$

$\alpha + \alpha r = 3$ & $\alpha r^2 + \alpha r^3 = 12$

$\alpha(1 + r) = 3$ & $\alpha r^2 (1 + r) = 12$

& $r^2 . 3 = 12$

$\therefore\ \alpha = \frac{3}{1+r}$ & $r^2 = 4$ $\qquad \Rightarrow \alpha = \frac{3}{3} \Rightarrow \alpha = 1$

$r = 2$,

$r = -2$ is rejected since

$\because$ G.P. increasing

$\alpha(\alpha r) = a$ & $(\alpha r^2)(\alpha r^3) = b$

$a = \alpha(\alpha r) = 1 (1.2) = 2$

$b = (\alpha r^2)(\alpha r^3) = (1.4)(1.8) = 32$

Q.15 (4)

Let GP be $a_1, a_2, ..., a_k$, ... with first term a & common ratio r,

$a_k = a_{k+1} + a_{k+2} \quad \forall\ a_k > 0$

$\Rightarrow ar^{k-1} = ar^k + ar^{k+1} \quad \Rightarrow \quad r > 0$

$\Rightarrow 1 = r + r^2 \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1+4}}{2}$ {r = –ve rejected}

$\Rightarrow r = \frac{\sqrt{5}-1}{2} = 2\left(\frac{\sqrt{5}-1}{4}\right) = 2 \sin 18^\circ$

Q.16 (1)

$S = \frac{p}{1-\frac{1}{p}} = \frac{9}{2}$

$\Rightarrow \frac{p^2}{p-1} = \frac{9}{2}$

$\Rightarrow\ 2p^2 - 9p + 9 = 0$

$\Rightarrow\ 2p^2 - 6p - 3p + 9 = 0$

$\Rightarrow\ (2p - 3)(p - 3) = 0$

$p = 3/2\ ,\ 3$

Q.17 (1)

Let $S = \sqrt{2} + \sqrt{6} + \sqrt{18} + \sqrt{54} + ... +$ (10 terms)

$= \sqrt{2}\ (1 + \sqrt{3} + \sqrt{9} + \sqrt{27} + ... +$ (10 terms))

$= \sqrt{2}\ (1 + 3^{1/2} + 3^1 + 3^{3/2} + ... +)$

$= \sqrt{2}\ \ 1. \frac{(1-(\sqrt{3})^{10})}{(1-\sqrt{3})} = \frac{\sqrt{2}((\sqrt{3})^{10}-1)}{(\sqrt{3}-1)}$

$= \frac{\sqrt{2}(3^5-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{2}}{2}\ 242\ (\sqrt{3} + 1)$

$= \sqrt{2}\ (121)(\sqrt{3} + 1) = 121(\sqrt{6} + \sqrt{2})$

Q.18 (1)

Let $S = \frac{1}{(1+p)} - \frac{1-p}{(1+p)^2} + \frac{(1-p)^2}{(1+p)^3} \ ... \infty$

$-1 < r = -\left(\frac{1-p}{1+p}\right) < 1 \qquad \because\ \ p > 0$

It is a GP with first term = $\frac{1}{1+p}$ & common ratio r

$\therefore\ S = \frac{a}{1-r} = \frac{\left(\frac{1}{1+p}\right)}{1+\frac{1-p}{1+p}} = \frac{1}{1+p+1-p} = \frac{1}{2}$

Q.19 (3)

If $x > 0$

$\log_2 x + \log_2(\sqrt{x}) + \log_2 \sqrt[4]{x} + \log_2 \sqrt[8]{x} + \log_2 \sqrt[16]{x} + ... = 4$

$\Rightarrow\ \log_2 x + \log_2 x^{1/2} + \log_2 x^{1/4} + ... = 4$

$\Rightarrow\ (\log_2 x)\ [1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...] = 4$

$\Rightarrow\ (\log_2 x) \frac{1}{1-\frac{1}{2}} = 4 \Rightarrow\ \log_2 x = 2 \Rightarrow\ x = 4$

Q.20 (1)

$3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... + \infty = 8$

$a = 3,\ r = \frac{1}{4}$

Sum of AGP upto ∞

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$$\Rightarrow\ 8 = \frac{3}{(3/4)} + \frac{d\left(\frac{1}{4}\right)}{3^2/4^2} \Rightarrow\ 8 = 4 + \frac{4d}{3^2}$$

$$\Rightarrow\ 4 = \frac{4d}{3^2} \Rightarrow\ d = 3^2 \quad \Rightarrow\ d = 9$$

Q.21 (3)

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} \Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{c^2} \Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow ab^2 + bc^2 = 2a^2c \Rightarrow \frac{b}{c} + \frac{c}{a} = \frac{2a}{b}$$

So $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P. $\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

Q.22 (1)

Given that,

$$x = \sum_{n=0}^{\infty} a^n,\ y \sum_{n=0}^{\infty} b^n,\ z = \sum_{n=0}^{\infty} c^n \quad \left\{\begin{array}{l} |a|<1 \\ |b|<1 \\ |c|<1 \end{array}\right.$$

$$x = \frac{1}{1-a},\ y = \frac{1}{1-b},\ z = \frac{1}{1-c}$$

∵ a, b, c are in A.P.

⇒ 1 – a, 1 – b, 1 – c are also in A.P.

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P. ⇒ x, y, z in H.P.

Q.23 (1)

a, b, c in A.P. ⇒ 2b = a + c ...(i)

p, q, r, in H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...(ii)

ap, bq, cr in G.P. $\Rightarrow b^2q^2 = acpr$

...(iii)

From (ii) & (iii), we get

$$\Rightarrow \frac{b^2 . 4(pr)^2}{(p+r)^2} = ac\ pr$$

$$\Rightarrow \frac{(a+c)^2 pr}{(p+r)^2} = ac \qquad \text{(from (i))}$$

$$\Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac} \Rightarrow \frac{p^2 + r^2}{pr} + 2 = \frac{a^2 + c^2}{ac} + 2$$

$$\Rightarrow \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}.$$

Q. 24 (3)

$a^x = b^y = c^z = d^t = k$ and a, b, c, d are in G.P.

a, b, c are in G.P. ⇒ So $b^2 = ac$

$$\Rightarrow k^{2/y} = k^{1/x + 1/z} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

⇒ x, y, z are in H.P.

∵ b, c, d are in GP

then $\frac{2}{z} = \frac{1}{y} + \frac{1}{t} \Rightarrow$ y, z, t are in HP

So x, y, z, t are in H.P.

Q.25 (2)

$$AM = A = \frac{a+b+c}{3}$$

$$GM = G = (abc)^{1/3}$$

$$HM = H = \frac{3abc}{ab+bc+ca} = \frac{3G^3}{ab+bc+ca}$$

Equation whose roots are a,b,c

$$\Rightarrow x^3 - (a + b + c)x^2 + (\Sigma ab)x - abc = 0$$

$\Rightarrow x^3 - 3Ax^2 + \frac{3G^3}{H} .x - G^3 = 0$ **Ans**

Q.26 (2)

$$\text{Let } S = \sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$$

$$= \sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)} = \frac{1}{2} \sum_{r=2}^{\infty} \left(\frac{1}{r-1} - \frac{1}{r+1}\right)$$

$$= \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + ... + \frac{1}{n-1} - \frac{1}{n+1} + ...\right]$$

when $n \to \infty \Rightarrow \frac{1}{n+1} \to 0$

$$\therefore S = \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{3}{4}.$$

Q.27 (3)

$$\text{Let } S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ... \infty = \frac{\pi^2}{6}$$

$$\text{Now } S_{even} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + ... \infty$$

$$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + ...\infty\right] = \frac{1}{2^2}\ \frac{\pi^2}{6} = \frac{\pi^2}{24}$$

$$S_{odd} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... \infty$$

$= S - S_{even}$

$$= \frac{\pi^2}{6} - \frac{\pi^2}{24} = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$$

Q.28 (1)

Let S = 1(1!) + 2 (2 !) + 3(3 !) + ... + n (n !)

$$\Rightarrow S = \sum_{r=1}^{n} r(r!) = \sum_{r=1}^{n} (r+1-1)r!$$

$$= \sum_{1}^{n}[(r+1)r! - r!]$$

= (n + 1) ! – 1

Q.29 (3)

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$n terms

$= \frac{n(n+1)^2}{2}$, when n is even

$1^2 + 2 . 2^2 + 3^2 + ... 2 . n^2 = n\ \frac{(n+1)^2}{2}$

when n is odd n + 1 is even

$1^2 + 2 . 2^2 + 3^2 + ... n^2 + 2 . (n + 1)^2$

$= (n + 1)\ \frac{(n+2)^2}{2}$

$1^2 + 2 . 2^2 + 3^2 + ... n^2 = (n + 1) \left[\frac{(n+2)^2}{2} - 2(n+1)\right]$

$= \frac{(n+1)\,n^2}{2}$

Q.30 (4)

Given $S = 2^n \cdot 2^{10}$

$S = 2.2^2 + 3.2^3 + 4.2^4 + ... + n.2^n$

$2S = 2.2^3 + 3.2^4 + ... + (n-1)2^n + n.2^{n+1}$

$\Rightarrow\ -S = 2^3 + 2^3 + 2^4 + ... + 2^n - 2.n^{n+1}$

$\Rightarrow -S = 1 + (1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^4) - n.2^{n+1}$

$\Rightarrow -S = 1 + \frac{1(2^{n+1}-1)}{2-1} - n.2^{n+1}$

$\Rightarrow -S = 1 + 2^{n+1} - 1 - n.2^{n+1}$

$\Rightarrow\ S = n.2^{n+1} - 2^{n+1} = 2^{n+1}(n-1) = 2^n . 2^{10}$

$\Rightarrow\ n - 1 = 2^9\ \Rightarrow\ n = 1 + 512$

$\Rightarrow\ n = 513$

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 (C)

common diff. = d, in A.P.

$T_7 = 9 \Rightarrow a + 6d = 9 \Rightarrow a = (9 - 6d)$

$T_1T_2T_7 = a . (a + d)\, 9 = (9 - 6d)(9 - 5d)\, 9$

$= 9\,(30d^2 - 99d + 81) = 27\,(10d^2 - 33d + 27)$

Min value at $d = \frac{-(-33)}{2\cdot 10} = \frac{33}{20}$

Q.2 (C)

If $a_1, a_2,..., a_n$ is AP with

$d = (a_2 - a_1) = (a_3 - a_2) = ... = (a_n - a_{n-1} \neq 0)$

sind [cosec a_1 cosec a_2 + coseca$_2$ cosec a_3 + ... + cosec a_{n-1} cosec a_n]

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} + ... + \frac{\sin d}{\sin a_{n-1} \sin a_n}$$

$$= \sum_{r=2}^{n} \frac{\sin(a_r - a_{r-1})}{\sin a_{r-1} \sin a_r}$$

$$= \sum_{r=2}^{n} \left(\frac{\sin a_r \cos a_{r-1}}{\sin a_{r-1} \sin a_r} - \frac{\cos a_r \sin a_{r-1}}{\sin a_{r-1} \sin a_r}\right)$$

$$= \sum_{r=2}^{n} (\cot a_{r-1} - \cot a_r) = \cot a_1 - \cot a_n$$

Q.3 (A)

Let $S_1 = \frac{n}{2}[2 + (n-1)1]$

$S_2 = \frac{n}{2}\ [4 + (n-1)\,3]$

$S_3 = \frac{n}{2}\ [6 + (n-1)5]$

- - - - - - - - - - - - - - - - -

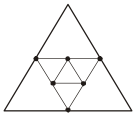
$S_p = \frac{n}{2}\ [2p + (n-1)(2p-1)]$

Thus, $S_1 + S_2 + S_3 + ... + S_p = \frac{n}{2}$ [(2 + 4 + 6 + ... + 2p)] + (n – 1) (1 + 3 + 5 + ... + (2p – 1)]

$= \frac{n}{2}\ [p\,(p + 1) + (n - 1)\,p^2]$

$= \frac{n}{2}\ [p^2 + p + np^2 - p^2] = \frac{np(1+np)}{2}$

Q.4 (A)



= 3[24 + 12 + 6 + ...∞]

$$= 3\ \frac{24}{1-\frac{1}{2}} = 144$$

Q.5 (C)

$$\frac{ar^{p-1}(r^n-1)}{r-1} = k . \frac{ar^{q-1}(r^n-1)}{r-1}$$

$r^{p-1} = k . r^{q-1}$

$k = r^{p-q}$.

Q.6 (C)

$$\frac{A_1 + A_2}{G_1G_2} = \frac{a+b}{ab}$$

Q.7 (D)

$P = \frac{a+b}{2}$, $\quad Q = \sqrt{ab}$

$$P - Q = \frac{a+b-2\sqrt{ab}}{2} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2}$$

Q.8 (C)

Let a, G_1, G_2, bin GP with first term a & common ratio r & a, A, b in AP

$\Rightarrow b = ar^3 \Rightarrow r = \left(\frac{b}{a}\right)^{1/3}$

$G_1 = ar = a\left(\frac{b}{a}\right)^{1/3}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/3} \qquad \because G_1G_2 = ab$

Now, $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3 + G_2^3}{G_1\,G_2} = \frac{a^3\frac{b}{a} + a^3\frac{b^2}{a^2}}{ab}$

$= \frac{a^2b + ab^2}{ab} = a + b = 2A \because 2A = a + b$

Q.9 (A)

Given that,

$H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$

Let $S = 1 + \frac{3}{2} + \frac{5}{3} + ... + \left(\frac{2n-1}{n}\right)$

$= \sum_{n=1}^{n}\left(\frac{2n-1}{n}\right) = \sum_{n=1}^{n}\left(2-\frac{1}{n}\right) = \sum_{n=1}^{n} 2 - \sum_{n=1}^{n}\frac{1}{n}$

$= 2n - \left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right) = 2n - H_n$

Q.10 (D)

Given that,

$x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x}+\frac{5}{y}+\frac{3}{z}\right)$

$\Rightarrow (x)^2 + (3y)^2 + (5z)^2 - 3xy - 15\,yz - 5zx = 0$

$\Rightarrow \frac{1}{2}\,[(x-3y)^2 + (3y-5z)^2 + (5z-x)^2] = 0$

$\Rightarrow x = 3y \;\&\; 3y = 5z \;\&\; 5z = x$

$\Rightarrow x = 3y = 5z \;\Rightarrow\; y = \frac{x}{3},\ z = \frac{x}{5}$

$x, y, z \Rightarrow x, \frac{x}{3}, \frac{x}{5}$

We know 1, 3, 5 in A.P. $\quad x \neq 0$

$\Rightarrow \frac{1}{x}, \frac{3}{x}, \frac{5}{x}$ in A.P. $\Rightarrow x, \frac{x}{3}, \frac{x}{5}$, in H.P.

$\Rightarrow$ x, y, z in H.P.

Q.11 (C)

$H = \frac{16}{5} = \frac{2ab}{a+b}$

...(i)

$A = \frac{a+b}{2}$, $G = \sqrt{ab}$

$\frac{16}{5} = \frac{2G^2}{2A} \Rightarrow 5G^2 = 16A$ ...(ii)

& Given $2A + G^2 = 26$

$\therefore\ 5(26 - 2A) = 16A \qquad$ (from (ii))

$\Rightarrow 5.26 = 26\,A \Rightarrow A = 5$

from (i) $\therefore\ a + b = 10$

$\frac{16}{5} = \frac{2ab}{10} \Rightarrow ab = 16 \Rightarrow a = 2, b = 8$

Q.12 (D)

Applying A.M. ≥ G.M.

$\frac{a+2b+3c+4d+5e}{15} \geq (a \cdot b^2 \cdot c^3 \cdot d^4 \cdot e^5)^{1/15}$

$\Rightarrow 5 \geq (a \cdot b^2 \cdot c^3 \cdot d^4 \cdot e^5)^{1/15}$

$\Rightarrow a \cdot b^2 \cdot c^3 \cdot d^4 \cdot e^5 = 5^{15}$

$\Rightarrow a^2 \cdot b^4 \cdot c^6 \cdot d^8 \cdot e^{10} = 5^{30}$

$\Rightarrow a^2 \cdot b^4 \cdot c^6 \cdot d^8 \cdot e^{10} \leq (125)^{10} \equiv (125)^p$

$\therefore\ p = 10$

Q.13 (B)

$E = \frac{8}{4\sin^3 x} + 3\sin^2 x = 2\operatorname{cosec}^3 x + 3\sin^2 x \leq 5$

using AM ≥ GM for 5 numbers cosec^3x, cosec^3x, $\sin^2x$, $\sin^2x$ and $\sin^2x$

we get

$2\operatorname{cosec}^3x + 3\sin^2x \geq 5$ ....(1)

but given $\quad 2\operatorname{cosec}^3x + 3\sin^2x \leq 5$ ....(2)

from (1) & (2) $\quad 2\operatorname{cosec}^3x + 3\sin^2x = 5$

which is possible for $x = \frac{\pi}{2} \qquad \Rightarrow \qquad (2)$

Q.14 (A)

$x_1 + x_2 + x_3 + ... + x_{50} = 50$ AM ≥ HM

$$\frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{1}{\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + ... + \frac{1}{x_{50}}}{50}}$$

$$\Rightarrow \frac{x_1 + x_2 + ... + x_{50}}{50}$$

$$\geq \frac{50}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} \geq 50$$

so min value of $\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} = 50$

Q.15 (A)

a, a_1, a_2 ...., a_{2n}, b are in AP

and

a, g_1, g_2, ...g_{2n}, b are in GP

and $h = \frac{2ab}{a+b}$

$$\therefore \frac{a_1 + a_{2n}}{g_1\ g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2\ g_{2n-1}} + ... + \frac{a_n + a_{n+1}}{g_n\ g_{n+1}}$$

$$= \frac{a+b}{ab} + \frac{a+b}{ab} + ... + \frac{a+b}{ab}$$

$$= 2n \cdot \left(\frac{a+b}{2ab}\right) = \frac{2n}{h}$$

Q.16 (B)

$$A = \frac{a+b+c}{3}, G = (a\ b\ c)^{1/3}, H = \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$H = \frac{3(abc)}{ab + bc + ca}$$

$x^3 - (a + b + c)\ x^2 + (ab + bc + ca)\ x - abc = 0$

$\Rightarrow\ x^3 - 3Ax^2 + (3G^3/H)x - G^3 = 0$

Q.17 $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...$

$$T_n = \frac{n}{1+n^2+n^2} = \frac{n}{n^4+n^2+1}$$

$$= \frac{1}{2}\left[\frac{(n^2+n+1)-(n^2-n+1)}{(n^2+n+1)\ (n^2-n+1)}\right]$$

$$S_n = \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + ... - \frac{1}{n^2+n+1}\right]$$

$$= \frac{1}{2}\left[\frac{n^2+n}{n^2+n+1}\right]$$

Q.18 (A)

Let S = 1 . (2 + 3 + 4 + ... + n) +

+ 2 (3 + 4 + 5 + ... + n) +

+ 3 (4 + 5 + ... + n) +

- - - - - - - - - - - - - - - - - - -

+ (n – 2) [(n + 1) + n]

+ (n – 1) (n)

S

$$= \frac{(1+2+3+4+...+n)^2 - (1^2+2^2+3^2+...+n^2)}{2}$$

$$= \frac{1}{2}\left[\frac{n^2(n+1)^2}{2^2} - \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{24} n(n + 1)\ [3n^2 + 3n - 4n - 2]$$

$= \frac{1}{24}$ n (n + 1) (3n^2 – n – 2) = $\frac{1}{24}$ n (n + 1) (n – 1) (3n + 2)

Aliter

1 . 2 + 1.3 + 1.4 + ... + 1. n

+ 2.3 + 2.4 + ... + 2. n

+ 3.4 + ... + 3. n

S = (1 . 2) + (1 + 2) 3 + (1 + 2 + 3) .4 + ...

+ [1 + 2 + 3 +.......+ (n – 1)]n

T_n = [1 + 2 + 3 + ... (n – 1)] n

$$= \Sigma \frac{n^2(n-1)}{2} = \frac{1}{2} (\Sigma n^3 - \Sigma n^2)$$

Q.19 (A)

Given that,

$1^2 + 2^2 + 3^2 + ... + 2003^2$ = (2003) (4007) (334)

& 1 (2003) + 2 (2002) + 3 (2001) + ... + 2003 (1)

= (2003) (334) (x)

$$\Rightarrow \sum_{r=1}^{2003} r[2003 - (r-1)] = (2003)\ (334)\ x$$

$$\Rightarrow \sum r[2004 - r] = (2003)\ (334)\ x$$

$$\Rightarrow 2004 \sum_{1}^{2003} r - \sum_{1}^{2003} r^2 = (2003)\ (334)\ x$$

$\Rightarrow \quad \frac{(2003)(2004)^2}{2} - (2003)\,(4007)(334)$

$= (2003)\,(334)\,x$

$\Rightarrow$ (2004) (1002) – (4007) (334) = (334)x

$\Rightarrow$ 6(1002) – (4007) = x

$\Rightarrow$ x = 6012 – 4007 = 2005

Q.20 (C)

$$\sum_{r=1}^{n} t_r = \frac{1}{12}\, n(n+1)\,(n+2) = S_n$$

$t_r = S_n - S_{n-1}$

$$= \frac{n(n+1)(n+2)}{12} - \frac{(n-1)n(n+1)}{12}$$

$$= \frac{n(n+1)}{12}\,[n + 2 - n + 1]$$

$$t_r = \frac{n(n+1)}{4} \Rightarrow \frac{1}{t_r} = \frac{4}{n(n+1)}$$

$$\sum_{r=1}^{n} \frac{1}{t_r} = 4\sum_{r=1}^{n} \frac{1}{n(n+1)} = 4 \sum\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= 4\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots + \frac{1}{n} - \frac{1}{n+1}\right] = \frac{4n}{n+1}$$

JEE-ADVANCED MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B,D)

(D) $a_1 + 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

a – 4(a + d) + 6(a + 2d) – 4(a + 3d) + (a + 4d)

= 0 – 0 = 0

Like wise we can check other options

Q.2 (A,B)

since x , |x + 1|, |x – 1| are in A.P.

so 2 |x + 1| = x + |x – 1| .... (i)

Case-I If x < – 1, then (i) becomes

$-2\,(x + 1) = x - (x - 1) \Rightarrow x = -\frac{3}{2}$

then series $-\frac{3}{2}, \frac{1}{2}, \frac{5}{2}$, ...

$\therefore \quad S_{20} = \frac{20}{2}\,[-3 + (20 - 1)\,2] = 350$

Case-II If $-1 \le x \le 1$, then (i) becomes

2 (x + 1) = x – (x – 1) $\Rightarrow$ x = – 1/2

then series $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

So $S_{20} = \frac{20}{2}\,[-1 + (20-1)\,1] = 10 \times 18 = 180$

Case-III If $x \ge 1$, then (i) becomes

2 (x + 1) = x + x – 1

2 = – 1 impossible.

Q.3 (B,D)

$a_1, a_2, \ldots, a_n, \ldots$ are in AP

$a_2 = \frac{a_1 + a_3}{2} \Rightarrow a_1 + a_3 - 2a_2 = 0$

& $a_1 - 2a_2 + a_3 = 0$ ...(i)

& $-2(a_2 - 2a_3 + a_4) = 0$ ...(ii)

& $a_3 - 2a_4 + a_5 = 0$ ...(iii)

Adding (i) (ii) & (iii), we get

$a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Q.4 (B,C)

Here, $A_1 = \frac{3}{2}$, $A_2 = \frac{7}{4}$, $A_3 = \frac{15}{8}$, $A_4 = \frac{31}{16}$

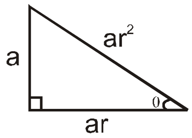
Here, $A_{10} = \frac{2047}{1024}$ and $\sum_{n=1}^{10} A_n > 19$.

Verify alternative.

Q.5 (B,C)

Case - I

r > 1



$a^2 + a^2r^2 = a^2r^4 \Rightarrow r^4 - r^2 - 1 = 0$

$r^2 = \frac{\sqrt{5}+1}{2}$

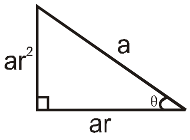
$r = \sqrt{\frac{\sqrt{5}+1}{2}}$ tangent of smallest angle = tanθ

$= \frac{1}{r} = \sqrt{\left(\frac{2}{\sqrt{5}+1}\right)}$

Case - II

0 < r < 1

$a^2 = a^2r^2 + a^2r^4 \Rightarrow r^4 + r^2 - 1 = 0$



$r^2 = \frac{\sqrt{5}-1}{2} \Rightarrow r = \sqrt{\frac{\sqrt{5}-1}{2}}$

tangent of smallest angle = tanθ = r = $\sqrt{\frac{\sqrt{5}-1}{2}}$

Q.6 (A,B,C,D)

b_1, b_2, b_3 are in G.P. ∴

$b_3 > 4b_2 - 3b_1 \Rightarrow r^2 > 4r - 3$

$\Rightarrow\ r^2 - 4r + 3 > 0$

$\Rightarrow\ (r-1)(r-3) > 0$

So $0 < r < 1$ and $r > 3$

Q.7 (A,C)

As, $a_1 + a_2 + a_3 = 126$

and $a_2 = \frac{a_1+a_3}{2} = \frac{126}{3} = 42.$

The numbers in A.P. are 42 – d, 42, 42 + d.

Let $g_1 = A,\ g_2 = AR;\ g_3 = AR^2$

$\therefore\quad AR = 34$ and $A + AR^2 = 85$

$\frac{34}{R} + 34R = 85 \Rightarrow\quad 34R^2 - 85R + 34 = 0$

$\Rightarrow\quad 34R^2 - 68R - 17R + 34 = 0$

$\Rightarrow\quad (R-2)(2R-1) = 0$

For $R = 2$, $A = 17$

$\therefore\quad g_1 = 43 + d = 17$

$\Rightarrow\quad d = -26$ (Rejected)

for $R = \frac{1}{2}$, $A = 68$

$\therefore\quad g_1 = 43 + d = 68 \Rightarrow\quad d = 25$

∴ Common difference of A.P. = 25 and common ratio of G.P. is $\frac{1}{2}$

Q.8 (B, C)

$$x = \sum_{n=0}^{\infty}\cos^{2n}\phi = \frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi},$$

$$y = \sum_{n=0}^{\infty}\sin^{2n}\phi = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$$

$$z = \sum_{n=0}^{\infty}\cos^{2n}\phi\ \sin^{2n}\phi = \frac{1}{1-\cos^2\phi\sin^2\phi}$$

$$z = \frac{1}{\left(1-\frac{1}{x}\cdot\frac{1}{y}\right)}$$

$\Rightarrow\ z = \frac{xy}{xy-1} \Rightarrow xyz - z = xy$

$\Rightarrow\ xyz = xy + z$

Since $xy = x + y \Rightarrow xyz = x + y + z$

Q.9 (A,B)

$2b = a + c$

$4b^2 = a^2 + c^2 + 2ac$

$\Rightarrow\ a^2 + c^2 = 4b^2 - 2ac$ & $b^2 = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow\ b^2(4b^2 - 2ac) = 2a^2c^2$

$\Rightarrow\ b^2(2b^2 - ac) = a^2c^2$

$\Rightarrow 2b^4 - b^2(ac) - (ac)^2 = 0$

$\Rightarrow (b^2 - ac)(2b^2 + ac) = 0$

$\Rightarrow\ b^2 = ac \Rightarrow$ a, b, c in G.P.

& a, b, c in A.P.

$\Rightarrow\ a = b = c$

or $2b^2 + ac = 0 \Rightarrow b^2 = -\frac{ac}{2}$

$\Rightarrow$ a, b, $\frac{-c}{2}$ in G.P.

Q.10 (A,C,D)

Let terms x, y, z be x, xr, xr^2

$\therefore\quad x + xr + xr^2 = 42$ .....(1)

and $\quad 2\cdot\frac{5}{4}(xr) = x + xr^2$

$\Rightarrow\quad 5r = 2 + 2r^2$

$\Rightarrow\quad 2r^2 - 5r + 2 = 0$

$\Rightarrow\quad (r-2)(2r-1) = 0$

$\therefore\quad r = 2$ or $r = \frac{1}{2}$

$\therefore\quad x = 6$ or $x = 24$

$\therefore\quad x_1 = 24,\ x_2 = 6$

sum of infinite G.P. $= \frac{24}{1-(1/4)} = 32$

and $\quad x_1 + x_2 = 30$

and $\quad \frac{\text{A.M.}}{\text{G.M.}} = \frac{30/2}{\sqrt{24\cdot 6}} = \frac{15}{12} = \frac{5}{4}.$

Q.11 **A,B,C**

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{2}{1}$$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$

use compendendo and dividendo rule

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\Rightarrow \frac{a}{b} = \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{4-3}$$

$= 7 + 4\sqrt{3}$ **Ans**

Q.12 (A,B,C

Given, $x = y + 2$, $a = 5z$

a, x, b in A.P $\Rightarrow$ $2x = a + b$

a, y, b in G.P $\Rightarrow$ $y^2 = ab$

a, z, b in H.P $\Rightarrow z = \dfrac{2ab}{a+b}$

$\Rightarrow z = \dfrac{2y^2}{2x} \Rightarrow y^2 = zx$

A.M. > G.M. > H.M.

$x > y > z$

$\dfrac{a}{5} = \dfrac{2ab}{a+b}$

$a^2 + ab = 10ab$

$\Rightarrow$ $a(a - 9b) = 0$

$\Rightarrow$ $a \neq 0$ or $a = 9b$

$y = x - 2$

$\therefore$ $y = \dfrac{2x-4}{2}$ $\Rightarrow$ $y^2 = \dfrac{(a+b-4)^2}{4}$

$\Rightarrow$ $a^2 + b^2 - 2ab - 8a - 8b + 16 = 0$

$\Rightarrow$ $4b^2 - 5b + 1 = 0$

$\Rightarrow$ $(b - 1)(4b - 1) = 0$

$\Rightarrow$ $b = 1, a = 9$

or $b = 1/4, a = 9/4$

Q.13 (A, B, C, D)

$$\sum_{r=1}^{n} r(r + 1)(2r + 3) = an^4 + bn^3 + cn^2 + dn + e$$

$$\sum_{r=1}^{n} (r^2+r)(2r + 3) \qquad = \sum_{r=1}^{n} (2r^3 + 5r^2 + 3r)$$

$$= 2.\frac{n^2(n+1)^2}{4} + 5\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{5}{3}(2n+1) + 3\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{6\left(n^2+n\right) + 10(2n+1) + 18}{6}\right]$$

$$= \frac{n(n+1)}{12}[6n^2 + 26n + 28]$$

$$= \frac{1}{12}[6n^4 + 26n^3 + 28n^2 + 6n^3 + 26n^2 + 28n]$$

$$= \frac{1}{12}[6n^4 + 32n^3 + 54n^2 + 28n]$$

$a = \dfrac{6}{12}$

$b = \dfrac{32}{12}$

$c = \dfrac{54}{12}$

$d = \dfrac{28}{12} = \dfrac{7}{3}$

$e = 0$

so $a + c = b + d$

$b - \dfrac{2}{3} = \dfrac{32}{12} - \dfrac{2}{3} = \dfrac{24}{12}$

$c - 1 = \dfrac{42}{12}$

so a, $b-2/3$, $c-1$ are in A.P

& $\dfrac{c}{a} = \dfrac{54}{6} = 9$ is an integer

Q.14 (A,C)

$$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$$

$$= \sum_{1}^{n} \frac{\sqrt{a+rx}-\sqrt{a+(r-1)x}}{x}$$

$$= \frac{1}{x}\sum_{1}^{n}\left(\sqrt{a+rx}-\sqrt{a+(r-1)x}\right)$$

$$= \frac{1}{x}\left[\sqrt{a+nx} - \sqrt{a}\right]$$

Upon rationalizing

$$= \frac{1}{x}\ \frac{a+nx-a}{\sqrt{a+nx}+\sqrt{a}} = \frac{n}{\sqrt{a+nx}+\sqrt{a}}$$

Q.15 (C,D)

$S_1 = 1 +$

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{2019} - 2$$
$$\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots\ldots + \frac{1}{2018}\right]$$

$$S_1 = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{2019}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{2009}\right)$$

$$S_1 = \frac{1}{1010}+\frac{1}{1011}+\frac{1}{1012}+\ldots\ldots+\frac{1}{2019} = S_2$$

$$S_1 + S_2 = 2S_2 = 2\left[\frac{1}{1010}+\frac{1}{1011}+\ldots\ldots+\frac{1}{2019}\right]$$

$<$ 2

$$\left(\frac{1}{1010}+\frac{1}{1011}+\ldots\ldots+\frac{1}{2019}\right)$$

$< 2 \times 1.$

Comprehension # 01 (Q. No. 16 to 18)

Q.16 (A)

$g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + (n-1)^2) = n^2$

Q.17 (A)

$$g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$$

$$\left(\frac{2n+1}{3}-1\right) = \frac{n(n+1)}{2}\ \frac{2n-2}{3}$$

$$= \frac{n(n+1)(n-1)}{3} = \frac{(n-1)\,n\,(n+1)}{3}$$

for n = 2 $\frac{(n-1)\,n\,(n+1)}{3} = \frac{1.2.3}{3}$ which is divisible by 2 but not by 2^2

∴ greatest even integer which divides $\frac{(n-1)\,n\,(n+1)}{3}$, for every $n \in N$, $n \geq 2$, is 2

Q.18 (D)

$$f(n) + 3g(n) + h(n) = \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + \left(\frac{n(n+1)}{2}\right)^2$$

$$= \frac{n(n+1)}{2}\left(1+2n+1+\frac{n(n+1)}{2}\right)$$

$$= (1 + 2 + 3 + \ldots\ldots + n)\left(2n+2+\frac{n(n+1)}{2}\right)$$

⇒ for all $n \in N$

Comprehension # 02 (Q. No. 19 to 21)

Q.19 (A)

Q.20 (C)

Q.21 (D)

19, 20, 21

Let the 3 numbers in strictly increasing GP are $\frac{a}{r}$, a, ar(r > 1)

According to passage $\frac{a^2}{r^2} + a^2 + a^2r^2 = S^2$

or $a^2\left(\frac{1}{r^2}+1+r^2\right) = S^2$

or $a^2\left(\frac{1}{r}+1+r\right)\left(\frac{1}{r}-1+r\right) = S^2$

...(i)

and $\frac{a}{r} + a + ar = \alpha S$

or $a\left(\frac{1}{r}+1+r\right) = \alpha S$

∴ $a^2\left(\frac{1}{r}+1+r\right)^2 = \alpha^2 S^2$ ..(ii)

Dividing Eq. (ii) by (i), then $\left(\frac{\frac{1}{r}+1+r}{\frac{1}{r}-1+r}\right) = \alpha^2$

⇒ $(1 + r + r^2) = \alpha^2\,(1 - r + r^2)$

⇒ $(\alpha^2 - 1)r^2 - (\alpha^2 + 1)r + \alpha^2 - 1 = 0$ …(iii)

(1) r = 3 ∴ r = $\frac{11}{7}$

(2) Put $\alpha^2 = 2$ in eq. (iii), then $r^2 - 3r + 1 = 0$

∴ $r = \frac{3 \pm \sqrt{5}}{2} = \frac{3+\sqrt{5}}{2}$ ∴ r > 2

(3) Put r = 2 in eq. (iii), then

$4(\alpha^2 - 1) - 2(\alpha^2 + 1) + a^2 - 1 = 0$

or $3\alpha^2 - 7 = 0$ ∴ $\alpha^2 = 7/3 = a/b$ ⇒ a + b = 10

Comprehension # 03 (Q. No. 22 to 24)

Q.22 (A)

Q.23 (B)

Q.24 (D)

Let 1st term be a . and common difference is 2

$T_{2n+1} = a + 4n = A$ (say) $r = \frac{1}{2}$

Middle term of AP = T_{n+1}

Middle term of GP = T_{3n+1}

$T_{n+1} = a + 2n \Rightarrow T_{3n+1} = A\,.\,r^n = \frac{(a+4n)}{2^n}$

$(a + 2n) = \frac{a+4n}{2^n} \Rightarrow$ $2^n\,a + 2n2^n = a + 4n$

$a = \frac{4n-2n.2^n}{2^n-1} \Rightarrow T_{2n+1} = a + 4n = \frac{4n-2n.2^n}{2^n-1} + 4n$

$$= \frac{2n\,.\,2^n}{2^n-1} = \frac{2^{n+1}n}{2^n-1}$$

$$T_{3n+1} = \frac{a+4n}{2^n} = \frac{2^{n+1}n}{2^n(2^n-1)} = \frac{2n}{2^n-1}$$

Comprehension # 04 (Q. No. 25 to 27)

Q.25 (C)
Q.26 (B)
Q.27 (A)

$$\underbrace{(11111...........111)}_{91 \text{ times}} = 1 + 10 + 10^2 + 10^3 + ... +$$

10^{90}

$$= \frac{1(10^{91}-1)}{(10-1)} \Rightarrow \frac{(10^{13\times7}-1)}{(10-1)}$$

$$\frac{((10^{13})^7-1)}{(10^{13}-1)} \times \frac{(10^{13}-1)}{(10-1)}$$

$$((10^{13})^6 + (10^{13})^5 + ...+1)$$

$$(10^{12} + 10^{11} + ...+1)$$

$$\frac{((10^7)^{13}-1)}{(10^7-1)} \times \frac{(10^7-1)}{(10-1)}$$

$$\Rightarrow \qquad ((10^7)^{12} + (10^7)^{11} + ...+1)$$

$$(10^6 + 10^5 + ...+1)$$

$\therefore$ when
$P > k_i$ p= 13, k_i

$$\sum k_i = \frac{6\times7}{2} = 21 \qquad \sum m_i = \frac{12\times13}{2} = 78$$

$\therefore$ $\sum m_i - \sum k_i$ = 78 – 21 = 57 = 19 × 3.

Ans.]

Sol.(26) $p < k$
Σk_i = 78 & Σm_i = 21

$\therefore$ cos (78 + 2 × 21)° = cos 125° = $-\frac{1}{2}$

Sol.(27) We know $(11)^2 = 121$
$(111)^2 = (1\ 2\ 3\ 2\ 1)$
$(1111)^2 = (1\ 2\ 3\ 4\ 3\ 2\ 1)$

$$\underset{91}{(11111......)^2} = (1\ 2\ 3\ 4\ ......\ 91 \cdot 90\ ......\ 3\ 2\ 1)$$

So $(1111\ ......\ 1)^2 - (1\ 2\ 3\quad 91 \cdot 90\ ......\ 3 \cdot 2 \cdot 1) = 0$]

Comprehension Type Questions # 5 (Q. No. 28 to 30)

Q.28 (C)
Q.29 (D)
Q.30 (A)

Sol.28. $\therefore$ function f (x) forms H.P.

29. $f(x) = \frac{1}{2x}$

30. $n > \theta$, $f(x) + f\left(\frac{1}{x}\right)$

$$\Rightarrow \quad \frac{1}{2x} + \frac{1}{2}(x)$$

$$\Rightarrow \quad \frac{1}{2}\left(x + \frac{1}{x}\right), \qquad x > 0$$

$$\Rightarrow \quad \frac{1}{2}(2)$$

(Ans. ≥ 4 AM)
$\Rightarrow$ 1

Q.31 (A) → (p), (B) → (p), (C) → (q), (D) → (q)

(A) $2\log_5(2^x - 5) = \log_5 2 + \log_5 (2^x - 7/2)$

$$(2^x - 5)^2 = 2\left(2^x - \frac{7}{2}\right)$$

$t^2 - 10t + 25 = 2t - 7$ {put $2^x = t$}
$t^2 - 12t + 32 = 0$
$\therefore$ t = 8 , 4
$\therefore$ $2^x = 4$ or $2^x = 8$
$\therefore$ x = 2 ,3 $2^x - 5 > 0$
$\therefore$ 2x = 6 $\therefore$ $2^x > 5$
so only solution x = 3

(B) $$\frac{S_{2n}}{S_n} = \frac{\frac{2n}{2}[2a+(2n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = 3$$

$$= \frac{2a+(2n-1)d}{2a+(n-1)d} = \frac{3}{2} \quad \therefore\ d = \frac{2a}{n+1}$$

Now $$\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]}$$

$$= \frac{3\left[2a+(3n-1)\frac{2a}{n+1}\right]}{\left[2a+(n-1)\frac{2a}{n+1}\right]} = \frac{3[(n+1)+(3n-1)]}{(n+1)+(n-1)}$$

$= \frac{3.4n}{2n} = 6$

(C) $S = 4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + ...\frac{4n}{3^{n-1}}$ ...(i)

$\frac{S}{3} = \frac{4}{3} + \frac{8}{3^2} + \frac{12}{3^3} + ...+ \frac{4m}{3^n}$ ...(ii)

(i) – (ii) we get

$\frac{2}{3}S = 4 + \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + ...+ \frac{4}{3^{n-1}} + \frac{4n}{3^n}$

$= 4\left(\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right) - \frac{4n}{3^n}$

$S = \frac{4\cdot 3}{2}\cdot\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^n\right) - \frac{4n}{3^n}\cdot\frac{3}{2}$

$\therefore S_\infty = 9$

(D) $\ell, \ell r, \ell r^2$

$\therefore \ell^3 r^3 = 27$ (volume)

$\ell r = 3$

surface area

$2(\ell . \ell r + \ell r . \ell r^2 + \ell r^2 . \ell) = 78$

$\ell^2 (r + r^2 + r^3) = 39 \Rightarrow \ell^2\left(\frac{3}{\ell} + \frac{3^2}{\ell^2} + \frac{3^3}{\ell^3}\right) = 39$

$3\ell + 3^2 + \frac{3^3}{\ell} = 39 \Rightarrow \ell + 3 + \frac{9}{\ell} = 13$

$\therefore \ell^2 - 10\ell + 9 = 0 \Rightarrow \ell = 1, 9$

$\Rightarrow \ell r = 3$ and $\ell > \ell r$

$\therefore r = \frac{1}{3} \therefore \ell = 9$

Q.32 (A) → (r), (B) → (p), (C) → (s), (D) → (p)

(A) $2(\log_z x)^2 = \log_x y . \log_y z$ G.P

$(\log_z x)^2 = \log_x z$

$(\log_z x)^3 - 1 = 0$

$\log_z x = 1, (\log_z x)^2 + \log_z x + 1 = 0$ (not possible)

$\therefore x = z$ ....(1)

$2y^3 = x^3 + z^3$ x^3, y^3, z^3 in A.P

$2y^3 = 2x^3$

$x = y$ ....(2)

$\therefore x = y = z$

Given $xyz = 64$,

$\therefore x = y = z = 4$

$\therefore \frac{3x}{y} = 3$

(B) $S_\infty = 2^{1/4} .4^{1/8} .8^{1/16}................\infty$

$S_\infty = 2^{1/4} .2^{2/8} .2^{3/16}................\infty$

$S_\infty = 2^{1/4 + 2/8 + 3/16 ....\infty} = 2^{S'_\infty}$

Let $S'_\infty = \frac{1}{4} + \frac{2}{8} + \frac{3}{16}............$

$S'_n = \frac{1}{4} + \frac{2}{8} + \frac{3}{16}............\frac{n}{2^{n+1}}$

...(i)

$\frac{S'_n}{2} = \frac{1}{8} + \frac{2}{16} + ...+ \frac{n-1}{2^{n+1}} + \frac{n}{2^{n+2}}$ ...(ii)

(i) – (ii) we get

$\frac{S'_n}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\frac{1}{2^{n+1}} - \frac{n}{2^{n+2}}$

$\frac{S'_n}{2} = \frac{1}{4}\left(\frac{1-(1/2)^n}{1-1/2}\right) - \frac{n}{2^{n+2}}$

$S'_n = \frac{2.2}{4}\left(1-\left(\frac{1}{2}\right)^n\right) - \frac{2n}{2^{n+2}}$

$S'_\infty = 1$

$\therefore S_\infty = 2^{S'_\infty} = 2$

(C) x,y,z are in A.P

$y = \frac{x+z}{2}$, or $2y = x + z$

$(x + 2y - z)(x + z + z - x)(z + x - y)$

$= (x + (x + z) - z)(x + z + z - x)(2y - y)$

$= 2x .2z .y = 4xyz$

$\therefore k = 4$

(D) $d = \frac{31-1}{m+1} = \frac{30}{m+1}$

$\frac{A_7}{A_{m-1}} = \frac{1+7\frac{30}{m+1}}{1+(m-1)\frac{30}{m+1}} = \frac{5}{9}$

$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$

$\Rightarrow 146\ m = 2044$

$\Rightarrow m = 14$

$\therefore \frac{m}{7} = 2$

NUMERICAL VALUE BASED

Q.1 (51)

Let first installment be = 'a' and the common difference of the A.P. be ‘d’

So $a + (a + d) + (a + 2d) + ... + (a + 39d) = 3600 \Rightarrow \frac{40}{2}[2a + 39d] = 3600$

$\Rightarrow \quad 2a + 39\,d = 180 \quad .....\ (1)$

and $\dfrac{30}{2}\ [2a + 29\,d] = 2400$

$\Rightarrow \quad 2a + 29d = 160$

..... (2)

By equations (1) & (2), we get

$d = 2$ and

$a = 51$ **Ans.**

Q.2 (1)

$$t_n = S_n - S_{n-1} = \frac{n}{6}\ (2n^2 + 9n + 13) - \frac{(n-1)}{6}$$

$\{2(n - 1)^2 + 9(n - 1) + 13\}$

$$= \frac{1}{6}\ [2n^3 + 9n^2 + 13n - 2(n - 1)^3 - 9\ (n - 1)^2 -$$

$13(n - 1)] = (n + 1)^2$

$$\sum_{r=1}^{\infty} \frac{1}{r\,.\,(r+1)} = \sum_{r=1}^{\infty}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1$$

Q.3 (20)

$\sqrt{a-x}\ ,\ \sqrt{x}\ ,\ \sqrt{a+x}$ are in A.P. then

$$2\sqrt{x} = \sqrt{a-x} + \sqrt{a+x}$$

$\Rightarrow \quad 4x = a - x + a + x + 2\sqrt{a^2 - x^2}$

$\Rightarrow \quad 4x = 2a + 2\sqrt{a^2 - x^2}$

$\Rightarrow \quad (2x - a) = \sqrt{a^2 - x^2}$

$\Rightarrow \quad 4x^2 + a^2 - 4ax = a^2 - x^2$

$\Rightarrow \quad 5x^2 = 4ax$

$\Rightarrow \quad x = \dfrac{4a}{5}$

If a and x should be integer then a = 20 is composite number.

$\therefore \quad a = 20.$ **Ans.**

Q.4 (12)

Sol. If $2\alpha^2, \alpha^4, 2r$ are in A.P. then

$2\alpha^4 = 2\alpha^2 + 24$

$\Rightarrow \quad \alpha^4 = \alpha^2 + 12$

$\Rightarrow \quad \alpha^4 - \alpha^2 - 12 = 0$

Then $\alpha^2 = \dfrac{1 \pm \sqrt{49}}{2}$

$\therefore \quad \alpha_1^{\,2} = \alpha_2^{\,2} = 4$

again $1, \beta^2, 6 - \beta^2$ are in G.P.

$(\beta^2)^2 = 1 \cdot (6 - \beta^2)$

$\Rightarrow \quad \beta^4 + \beta^2 - 6 = 0$

$\therefore \quad \beta^2 = \dfrac{-1 \pm \sqrt{25}}{2}$

$\Rightarrow \quad 2, -3$

$\beta_1^{\,2} = \beta_2^{\,2} = 2$

$\therefore \quad \alpha_1^{\,2} + \alpha_2^{\,2} + \beta_1^{\,2} + \beta_2^{\,2} = 4 \times 2 + 2 \times 2 = 12.$ **Ans.**

Q.5 (8)

Let AP has 2n terms

Sum of odd term = 24

$\Rightarrow \quad \dfrac{n}{2}\ [a_1 + a_{2n-1}] = 24$

..... (1)

and sum of even terms = 30

$\Rightarrow \quad \dfrac{n}{2}\ [a_2 + a_{2n}] = 30 \quad .....\ (2)$

and $\quad a_{2n} = a_1 + \dfrac{21}{2}$

$a_1 + (2n - 1)d = a_1 + \dfrac{21}{2}$

$\Rightarrow \quad (2n - 1)\ d = \dfrac{21}{2} \quad .....\ (3)$

By equation (1) & (2)

$a_1 + a_{2n-1} = \dfrac{48}{n}$ and $a_2 + a_{2n} = \dfrac{60}{n}$

So $\quad a_1 + (n - 1)d = \dfrac{24}{n}$

and $a_1 + nd = \dfrac{30}{n}$ So $d = \dfrac{6}{n}$

Now $(2n - 1)\ \dfrac{6}{n} = \dfrac{21}{2} \Rightarrow n = 4,$

$d = \dfrac{6}{4} = \dfrac{3}{2}$

So no. of terms = 2n = 8 and $a_1 = 3/2$.

Numbers are $\dfrac{3}{2}, 3, \dfrac{9}{2}$ ....

Q.6 (50)

$x_1 + x_2 + x_3 + ... + x_{50} = 50$

$AM \geq HM$

$$\frac{x_1 + x_2 + ... + x_{50}}{50}$$

$$\geq \frac{1}{\dfrac{\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3} + ... + \dfrac{1}{x_{50}}}{50}}$$

$\Rightarrow \quad \dfrac{x_1 + x_2 + ... + x_{50}}{50}$

$$\geq \frac{50}{\frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}} \geq 50$$

so min value of $\frac{1}{x_1}+\frac{1}{x_2}+........+\frac{1}{x_{50}} = 50$

Q.7 **(64)**

$$\frac{(a_1+a_2)+(a_3+a_4)}{2} \geq \sqrt{(a_1+a_2)(a_3+a_4)} \quad ;$$

$$(a_1+a_2)(a_3+a_4) \leq 64$$

Q.8 (2)

ar^2, $3a.ar^3$, ar are in A.P. $d = 1/8$

$3a^2r^3 - ar^2 = 1/8$ .....(1);

$ar = a^2 + 2.1/8$ ......(2)

from (2) $a = (-)^2 = \frac{1}{4r(1-r)}$

......(3)

from (1) & (3) $r = \frac{1}{2}$; $r = -2$

but $0 < r < 1$

$r = \frac{1}{2} \Rightarrow a = 1 \Rightarrow 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ .... sum $= \frac{1}{1-\frac{1}{2}}$

= 2

Q.9 (0)

a, b, c are in G.P.

$\Rightarrow \quad b^2 = ac$

$\Rightarrow (a - b), (c - a), (b - c)$ are in H.P.

So $\frac{1}{a-b}, \frac{1}{c-a}, \frac{1}{b-c}$ are in AP.

Let a, b, c are $\frac{b}{r}$, b, br

So $\frac{1}{\frac{b}{r}-b}, \frac{1}{br-\frac{b}{r}}, \frac{1}{b-br}$ are in AP.

So $\frac{2}{br-\frac{b}{r}} = \frac{1}{\frac{b}{r}-b} + \frac{1}{b-br}$

$$\Rightarrow \frac{2r}{r^2-1} = \frac{r}{1-r} + \frac{1}{1-r}$$

$$\Rightarrow -\frac{2r}{(r+1)} = (1 + r)$$

$$\Rightarrow (1 + r)^2 = -2r$$

$$\Rightarrow r^2 + 1 + 4r = 0 \quad \Rightarrow \frac{c}{a} + 1 + \frac{4b}{a} = 0$$

$\Rightarrow a + 4b + c = 0$

Q.10 (0)

$\frac{a+b}{2} = 6$

$G^2 + 3H = 48$

$\Rightarrow ab + 3\frac{2ab}{a+b} = 48 \quad \Rightarrow ab + \frac{3ab}{6} = 48$

$\Rightarrow \frac{3}{2}ab = 48 \quad \Rightarrow ab = 32$

$\Rightarrow a = 4, b = 8.$

Q.11 **(65)**

$$S = \frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + ...\infty$$

$$S = \frac{5}{9}\left[\frac{(10-1)}{13} + \frac{10^2-1}{(13)^2} + \frac{10^3-1}{(13)^3} + ...\infty\right]$$

$$= \frac{5}{9}\left[\frac{\frac{10}{13}}{1-\frac{10}{13}} - \left(\frac{\frac{1}{13}}{1-\frac{1}{13}}\right)\right] = \frac{5}{9}\left[\frac{10}{3} - \frac{1}{12}\right]$$

$$= \frac{5}{9}\left[\frac{39}{12}\right] = \frac{65}{36}$$ **Ans**

Q.12 **(54)**

$$S = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + ...\infty$$

...(i)

$$-\frac{1}{5}S = -\frac{1}{5} + \frac{2^2}{5^2} - \frac{3^2}{5^3} + \frac{4^2}{5^4} - \frac{5^2}{5^5} + ...\infty$$

...(ii)

(i) – (ii) we get

$$\frac{6}{5}S = 1 - \frac{3}{5} + \frac{5}{5^2} - \frac{7}{5^3} + \frac{9}{5^4} - \frac{11}{5^5} + ...\infty$$

...(iii)

$$-\frac{6}{25}S = -\frac{1}{5} + \frac{3}{5^2} - \frac{5}{5^3} + \frac{7}{5^4} - \frac{9}{5^5} + ...\infty$$

...(iv)

(iii) – (iv) we get

$$\frac{6s}{5}\left(\frac{6}{5}\right) = 1 - \frac{2}{5}+\frac{2}{5^2}-\frac{2}{5^3}+\frac{2}{5^4} - ....\infty$$

$$\frac{36}{25}\,S = 1 - \frac{2}{5}\left[\frac{1}{1+\frac{1}{5}}\right] \;;\; \frac{36}{25}\,S = 1 - \frac{2}{5}\left(\frac{5}{6}\right) = \frac{2}{3}$$

$$S = \frac{25}{36}\times\frac{2}{3} = \frac{25}{54} \quad \textbf{Ans}$$

Q.13 (9)

$$\frac{S_3(1+8S_1)}{S_2^2} = \frac{\left[\frac{n(n+1)}{2}\right]^2\left[1+\frac{8n(n+1)}{2}\right]}{\left[\frac{n(n+1)(2n+1)}{6}\right]^2}$$

$$= \frac{[1+4n(n+1)]9}{(2n+1)^2} = 9 \quad \textbf{Ans}$$

Q.14 (7)

$$T_n = \frac{n}{1+n^2+n^4} = \frac{1}{2}\left[\frac{(2n)}{(1+n+n^2)(1-n+n^2)}\right] ;$$

$$T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2}-\frac{1}{1+n+n^2}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]$$

$$T_3 = \frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right],$$

$$\vdots$$

$$T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2}-\frac{1}{1+n+n^2}\right]$$

$$S_n = \sum T_n = \frac{1}{2}\left[1-\frac{1}{1+n+n^2}\right] = \frac{n+n^2}{2(1+n+n^2)}$$

KVPY

PREVIOUS YEAR'S

Q.1 (C)

$$\frac{2^2+1}{2^2-1}+\frac{3^2+1}{3^2-1}+\frac{4^2+1}{4^2-1}+......+\frac{(2011)^2+1}{(2011)^2-1}$$

$$\sum_{r=2}^{2011}\frac{r^2+1}{r^2-1} = \sum_{r=2}^{2011}\left[1+\frac{2}{(r+1)(r-1)}\right]$$

$$= \sum_{r=2}^{2011}\left[1+\frac{1}{r-1}-\frac{1}{r+1}\right]$$

$$= 2010+\left[1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+.....+\frac{1}{2010}-\frac{1}{2012}\right]$$

$$= 2010+1+\frac{1}{2}-\frac{1}{2012}-\frac{1}{2011}$$

$$= 2011+\frac{1}{2}-\left[\frac{1}{2011}+\frac{1}{2012}\right]$$

lies between $\left(2011, 2011\frac{1}{2}\right)$

Q.2 (C)

$$\frac{x}{\frac{2n(2n+1)(4n+1)}{6}-x} < 1.01$$

$$2.01 < (1.01)\ \frac{2n(2n+1)(4n+1)}{6}$$

$$2.01.\frac{4n(n+1)(2n+1)}{6} < (1.01)\ \frac{2n(2n+1)(4n+1)}{6}$$

$$\frac{2.01}{1.01} < \frac{4n+1}{2n+2} \Rightarrow n > 150.5$$

Q.3 (C)

Let roots α – d, α, α + d

Product

Sum 3α = – ⇒ a = – $\frac{a}{3}$

$\alpha(\alpha^2 - d^2) = -a$

pair product $b = \alpha^2 - \alpha d + \alpha^2 + \alpha d + \alpha^2 - d^2$

$\alpha^2 - d^2 = 3$

$b = 2\alpha^2 + 3$

$b - 3 = \frac{2}{9}$ a2 ⇒ locus $x^2 = \frac{9}{2}(y-3)$ parpbola

Q.4 (C)

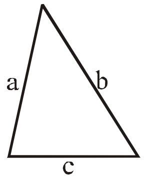
$\frac{x+y}{2} = 10a + b,\ \sqrt{xy} = 10b + a$

(a,b ∈ N)

$xy = (10b + a)^2$

$(x - y)^2 = 4\ (11a + 11b)\ (9a - 9b)$
$= 4 . 11 .(a + b). 9\ (a - b)$
$\Rightarrow a + b = 11\ ,\ a - b = 1$
$a = 6,\ b = 5$
($(x - y)^2$ is perfect square of a integer)
$x + y = 130$

Q.5 (B)



as a, b, c are an AP so 10, 10 + d , 10 + 2d
sum of a + b > c
20 + d > 10 + 2d
10 > d
As the d is minimum, Hence total possibility of d is 9

Q.6 (D)

$$\frac{2^2\left(1^2+2^2+3^2+......+n^2\right)}{\left[1^2+2^2+3^2+.....+(2n)^2\right]-2^2(1^2+2^2+3^2+.....n^2)}$$

$$\frac{\frac{2^2 n\left(n+1\right)\left(2n-1\right)}{6}}{\frac{2n\left(2n+1\right)\left(4n+1\right)}{6}-\frac{2^2\,n\left(n+1\right)\left(2n+1\right)}{6}}>1.0$$

$$\frac{2n+2}{2n-1}>1.01$$

$$1+\frac{3}{2n-1}>1.01$$

$2n - 1 > 300$
$n < 150.5$

Q.7 (D)

$$\left(\frac{a\sqrt{2}+b}{b\sqrt{2}+c}\right)\left(\frac{b\sqrt{2}-c}{b\sqrt{2}-c}\right)$$

$$=\frac{2ab-\sqrt{2}ac+\sqrt{2}b^2-bc}{2b^2-c^2}=\frac{b\left(2a-c\right)+\sqrt{2}\left(b^2-ac\right)}{2b^2-c^2}$$

Is rational when $b^2 = ac$ i,e. a, b, c are in GP
Here given option (A),(B), (C) does not satisfy the criteria But option (D) always satisfy

Q.8 (D)

$a_1, a_2, a_3, …….,a_{2012} = 3018$…………………(1)

$$\frac{a_1+a_3}{2}=a_2$$

$2a_2 + 2a_4 + 2a_6 + ...+ 2a_{2012} = 6036$
$(a_1 + a_3) + (a_3 + a_5) + (a_5 + a_7) + ...+ (a_{2011} + a_1) = 6036$
$2(a_1 + a_3 + a_5 +......+a_{2011}) = 6036$
$a_1 + a_3 + a_5 +......+a_{2011} = 3018$ ....(2)
Add (1) and (2)
Sum of all number = 3018 + 3018 = 6036

Q.9 (B)

$a_1 = 5$
$a_n = a_1a_2 ….. a_{n-1} + 4$
$a_2 = a_1 + 4 = 9$
$a_3 = a_1.a_2 + 4 = 5 \times 9 + 4 = 49$
$a_4 = a_1a_2a_3 + 4 = 2209$
$a_5 = a_1a_2a_3a_4 + 4 = 4870849= (2207)^2$
$a_5 = (a_4 - 2)^2$
$a_4 = (49 - 2)^2 = (a_3 - 2)^2$
$a_3 = (9 - 2)^2 = (a_2 - 2)^2$
$a_n = (a_{n-1} - 2)^2$

$$\sqrt{a_n}=a_{n-1}-2$$

$$\frac{\sqrt{a_n}}{a_{n-1}}=\frac{a_{n-1}-2}{a_{n-1}}=1-\lim_{x\to\infty}\frac{2}{a_{n-1}}$$

$$\because \lim_{x\to\infty}a_{n-1}=\infty$$

$= 1$

Q.10 (B)

$a = C - 2D$
$b = C - D$
$c = C$
$d = C + D$
$e = C + 2D$
$a + b + c+ d + e = 5c = \lambda^3$
$b + c + d = 3c = \mu^2$

$$\Rightarrow 3\lambda^3=5\mu^2$$

$$\frac{\lambda^3}{5}=\frac{\mu^2}{3}\text{least possibility}$$

$\lambda = 5 \times 3,\ \mu = 5 \times 3 \times 3$
$\lambda = 15\ \mu = 45$

$$C=\frac{\left(45\right)^2}{3}=15\times 45=675$$

Q.11 (C)

Taking three no’s x + 1, y + 1, z + 1
AM ≥ GM.

$$\frac{(x+1)+(y+1)+(z+1)}{3} \geq \{(x + 1)\ (y + 1)\ (z + 1)\}^{1/3}$$

$$\left(\frac{13}{3}\right)^3 \geq xyz + xy + yz + zx + 11$$

$$\left(\frac{13}{3}\right)^3 - 11 \geq xyz + xy + yz + zx$$

equality hold when x = y = z but x + y + z = 0 and x, y, z are integers. So maximum value when any two of x, y, z are equal to 3 and third is equal to 4

Q.12 (A)

$$\sum_{k=1}^{n}(ak^3+bk^2+ck+d)=n^4$$

$$a\sum_{k=1}^{n}k^3+b\sum_{k=1}^{n}k^2+c\sum_{k=1}^{n}k+\sum_{k=1}^{n}d=n^2$$

n^4 (12 –3a) –n^3 (4b + 6a) –n^2 (6c + 6b + 3a) – n(6c + 2b + 12d) = 0

12 –3a = 0, 4b + 6a = 0,

6c +6b + 3a = 0, 6c + 2b + 12d = 0

$\Rightarrow$ a = 4, b = –6, c = 4, d = –1

|a|+|b|+|c|+|d| = 15

Q.13 (B)

$$\frac{a}{4}=\frac{9}{b} \Rightarrow ab = 36$$

using AM $\geq$ GM

$$\frac{a+b}{2}\geq\sqrt{ab} \Rightarrow a + b \geq 12$$

Q.14 (A)

$$\frac{\left(\frac{2n(2n+1)}{2}\right)^2}{\frac{n(n+1)(2n+1)}{6}}=\frac{6n^2(2n+1)^2}{n(n-1)(2n+1)}$$

$$=\frac{6n(2n+1)}{(n+1)}=\frac{6n(n+1+n)}{n+1}$$

$$=6n+\frac{6n^2}{n+1}=6n+6(n-1)+\frac{6}{n+1}=\text{Integer}$$

The values of ‘n’ which are satisfying are n = 1, 2, 5 only for being integer

$\therefore$ sum = 1 + 2 +5 =8

Q.15 (C)

Let maximum house is ‘n’; sum of first ‘n’ even natural numbers = n^2 + n

Let first ‘m’ even natural numbers are left in numbering the houses.

(n^2 + n) – (m^2 + m) = 170

$\Rightarrow$ $n^2 - m^2 + n - m = 170$

$\Rightarrow$ (n – m) (n + m + 1) = 170

n – m = 10 $\Rightarrow$ n – m = 10

n + m + 1 = 17 $\Rightarrow$ n + m = 16

n = 13; m = 3

n $\leq$ 13

If first term is a– 10 then sixth = a

$$\frac{n}{2}[2(a-10)+2(n-1)]=170$$

$$\Rightarrow n[a+n-11]=170$$

$$\Rightarrow a=\frac{170}{n}+11-n$$

$\therefore$ n = 10 (only will make ‘a’ integer)

$$\Rightarrow a=17+11-10=18$$

Q.16 (A)

$$\sum_{n=0}^{1947}\frac{1}{2^n+\sqrt{2^{1947}}}$$ Total terms =1948

$$T_1=\frac{1}{1+\sqrt{2^{1947}}}$$

$$T_{1948}=\frac{1}{2^{1947}+\sqrt{2^{1947}}}$$

$$T_1+T_{1948}=\frac{1}{\sqrt{2^{1947}}}$$

Similarly, $T_2 + T_{1947} = \frac{1}{\sqrt{2^{1947}}} = T_3 + T_{1946} =$ and so on........

Total $\frac{1948}{2}=974$ pairs

$$\therefore \text{Sum}=\frac{974}{\sqrt{2^{1947}}}=\frac{974}{\sqrt{4\times2^{1945}}}=\frac{487}{\sqrt{2^{1945}}}$$

Q.17 (C)

n + 2n + 3m + ... + 99n + k^2

$\Rightarrow$ $n\frac{99.100}{2}=k^2$

$\Rightarrow$ n.99.50 = k^2

$\Rightarrow$ n.9. 11.25.2 = k^2

So n = 11.2 = 22

n^2 = 484

No. of digit in n^2 = 3.

Q.18 (B)

$$(I)\frac{x^3+y^3+z^3}{3}=\left(x^3y^3z^3\right)^{1/3}$$

Hence x = y = z {AM = GM}

$$(II)\frac{x^3+y^2z+yz^2}{3}\geq(x^3y^3z^3)^{1/3} \text{ (AM = GM)}$$

(III) $x^3 + y^2z + z^2x = 3xyz$

$$\frac{x^3\ \frac{y^2z}{2}+\frac{y^2z}{2}+z^2x}{4}\geq\left(\frac{x^4y^4z^4}{4}\right)^{1/4}$$

$$\Rightarrow\frac{3xyz}{4}\geq\frac{(xyz)}{\sqrt{2}} \text{ Not possible}$$

(IV) $\frac{x+y+z}{3}\geq(xyz)^{1/3} \Rightarrow (x + y + z)^3 = 27$ xyz.

Q.19 (C)

$2\{(2014)^3 + (2012)^2 +......2^3\} - \{(2014)^3 + (2013)^3 +....+...1^3\}$

$2 \times 8\{(1007)^2 + (1006)^2 +......1^3\} - \{(2014)^3 + (2013)^3 +....+...1^3\}$

$$=2\times8\left(\frac{(1007)(1008)}{2}\right)^2-\left(\frac{(2014)(2015)}{2}\right)^2$$

$= 2\times 8\times \frac{(1007)^2(1008)^2}{4} - \frac{(2014)^2(2015)^2}{4}$

$= (1007)^2\ (1016)^2 - (1007)^2\ (2015)^2 = (1007)^2\ \{2016 - 2015\}\ \{2016 + 2015\}$

$= (1007)^2\ (4031)$ = divisible by $(1007)^2$

Q.20 (B)

$a + 2b \le 1$

$A_1 = \pi a^2 b^6$

$A_2 = \pi b^4$

$\Rightarrow \frac{A_1}{A_2} = a^2b^2$

& $1 \ge a + 2b \ge \sqrt{2ab}$ (AM ≥ GM)

$\Rightarrow 1 \ge 2\sqrt{2ab}$

$\Rightarrow 1 \ge 4\cdot 2ab \qquad \Rightarrow \frac{1}{64} \ge a^2b^2$

Q.21 (A)

Note that for every real number a_i

$a_i.2^{a_i} > a_i$ and $a_i.2^{-a_i} < a_i$

therefore, $\sum_{i=1}^{100} a_i.2^{a_i} > \sum_{i=1}^{100} a_i$ and $\sum_{i=1}^{100} a^i.2^{-a_i} > \sum_{i=1}^{100} a_i$

Q.22 (A)

the possibility is

$A_1| = 1\ ;\ |A_2| = 2\ ;\ |A_3| = 3\ ;..........., |A_m| = m$

$1 + 2 + 3 + ... + m \le 100$ { Because all are disjoint}

$\Rightarrow \frac{m(m+1)}{2} \le 100 \Rightarrow m < 14$

14th set will have the same size as that of one of the previous sets. So, m = 13

Q.23 (A)

$$S_1 = a^2 = \frac{a^2}{4} + \frac{a^2}{16} + ...\infty = \frac{a^2}{1-\frac{1}{4}} = \frac{4a^2}{3}$$

$$S_2 = \frac{a^2}{2} + \frac{a^2}{8} + \frac{a^2}{32} + ...\infty \frac{2}{1-\frac{1}{4}} = \frac{4a^2}{6}$$

$\therefore \frac{S_1}{S_2} = 2$

Q.24 (D)

Given : $\frac{\ell_1}{\ell_2} + \frac{\ell_2}{\ell_3} .... + \frac{\ell_n}{\ell_1} = n$ ......(i)

$\therefore$ Use A.M ≥ G.M

We get

$$\frac{\left(\frac{\ell_1}{\ell_2} + \frac{\ell_2}{\ell_3} .... + \frac{\ell_n}{\ell_1}\right)}{n} \ge \sqrt[n]{\frac{\ell_1}{\ell_2}.\frac{\ell_2}{\ell_3}........\frac{\ell_n}{\ell_1}}$$

$\therefore \frac{n}{n} \ge 1$

$\Rightarrow n = n$

So A.M = G.M

Hence $\frac{\ell_1}{\ell_2} = \frac{\ell_2}{\ell_3} .... = \frac{\ell_n}{\ell_1} = k$

$\Rightarrow k = \frac{\ell_1 + \ell_1 .... + \ell_n}{\ell_2 + \ell_3 ..... + \ell_n + \ell_1} = 1$

$\Rightarrow \ell_1, \ell_2, ..., = \ell_n$

Q.25 (B)

(x, y, z) are real & x^4, y^4, z^4 are positive real numbers

$\therefore \frac{x^4 + y^4 + z^4 + 1}{4} \ge |xyz| \Rightarrow (xyz) \ge |xyz|$

i.e., xyz > 0

So it holds equality

$\therefore x^4 = y^4 = z^4 = 1$; But xyz > 0

$\therefore$ (x, y, z) $\in$ {(1, 1, 1)(1,–1,–1), (–1, 1, –1)(–1, –1, 1)}

So no. of triplets is 4.

Q.26 (A)

q > 0, try to the contra prove that q < $\frac{22-p}{21}$

$q + \frac{p}{21} < \frac{22}{21}$

$$q + \frac{p}{21} = \frac{1}{20}\left[\sum_{i=1}^{20}\frac{1}{a_i} + \frac{1}{21}\sum_{i=1}^{20} a_i\right]$$

$$= \frac{1}{20}\left[\sum_{i=1}^{20}\left(\frac{i}{i^2+1}\right) + \frac{1}{21}\sum_{i=1}^{20}\left(i + \frac{1}{i}\right)\right]$$

$$= \frac{1}{2} + \frac{1}{20}\left[\sum_{i=2}^{20}\frac{i}{i^2+1} + \sum_{i=1}^{20}\frac{1}{21i}\right]$$

$$< \frac{1}{2} + \frac{1}{20}\left[\frac{1}{2} + \frac{2}{5}\sum_{i=2}^{20} 1 + \frac{1}{21}\sum_{i=1}^{20} 1\right]$$

$$< \frac{1}{2} + \frac{1}{20}\left[\frac{1}{2} + \frac{2}{5}\times 19 + \frac{1}{21}\times 20\right]$$

$$< \frac{1}{2} + \frac{1}{20}(1+8+1) \quad < \frac{1}{2} + \frac{1}{2} \quad < \frac{22}{21}$$

Q.27 (A)

If x_1, x_2, ...x_n be n numbers the using Cauchy - Schwarz theorem -

$$\left(\frac{x_1 + x_2 + ...x_3}{n}\right)^2 \le \left(\frac{x_1^2 + x_2^2 + ...x_n^2}{n}\right)$$

Case (i) Consider

$$x_1 + x_2 + x_2 + x_3 + x_3 + x_3 .... + \underbrace{x_{2018} + x_{2018} + x_{2018}}_{\text{2018 times}}$$

Now using Cauchy-schwarz for above numbers

$$\left(\frac{x_1 + x_2 + x_2 + \ldots \dfrac{x_{2018} + x_{2018} + \ldots x_{2018}}{2018\text{times}}}{1 + 2 + 3 + \ldots..2018}\right)^2$$

$$\le \frac{x_1^2 + x_2^2 + x_2^2 + \ldots \dfrac{x_{2018}^2 + x_{2018}^2 + \ldots}{2018\text{times}}}{1 + 2 + 3 + \ldots..2018}$$

$$\Rightarrow \left(\frac{x_1 + 2x_2 + 3x_3 + \ldots 2018x_{2018}}{\sum\limits_{k=1}^{2018} k}\right)^2 \le \left(\frac{x_1^2 + 2x_2^2 + 3x_3^3 + \ldots 2018x_{2018}^2}{\sum\limits_{k=1}^{2018} k}\right)^2$$

$$\Rightarrow \left(\sum_{k=1}^{2018} kx_k\right)^2 \le \sum_{k=1}^{2018} k \left(\sum_{k=1}^{2018} kx_{k^2}\right)$$

$$\left(\sum_{k=1}^{2018} kx_k\right)^2 \le N\left(\sum_{k=1}^{2018} kx^2_{k^2}\right)$$

Therefore statement 1 is True.

Case (ii) Consider

$x_1 + 2x_2 + 3x_3 + \ldots..2018x_{2018}$

Now apply chauchy - Schwarz for above number

$$\left(\frac{x_1 + 2x_2 + 3x_3 + \ldots.2018x_{2018}}{2018}\right)^2$$

$$\le \frac{x_1^2 + 2x_2^2 + \ldots.(2018x_{2018})^2}{2018}$$

$\Rightarrow (x_1 + 2x_2 + \ldots..2018_{x2018})^2 \le 2018$ $\left(x_1^2 + 4x_2^2 + \ldots.(2018)^2 x_{2018}^2\right)$

$$\Rightarrow \left(\sum_{k=1}^{2018} kx_k\right)^2 \le 2018\left(\sum_{k=1}^{2018} k^2 x_k^2\right)$$

Since $n = \left(\sum\limits_{k=1}^{2018} kx_k\right)^2 = \dfrac{2018 \times 2019}{2}$

$\because \left(\sum\limits_{k=1}^{2018} kx_k\right)^2$ is always less than or equal to 2018 $\sum\limits_{k=1}^{2018} k^2 x_k^2$

$\therefore$ It will always be less than N $\left(\sum\limits_{k=1}^{2018} k^2 x_k^2\right)$

Q.28 (C)

Appling Am $\ge$ gm.

$$\frac{x^4 + 4y^4 + 16z^4 + 64}{4} \ge \left(4^6 x^4 y^4 z^4\right)^{1/4}$$

$x^4 + 4y^4 + 16z^4 + 64 \ge 32\ |\ xyz\ |$

so equal when each term is equal.

$\therefore$ $x^4 = 4y^4 = 16z^4 = 64$

$\Rightarrow$ $x = \pm 2\sqrt{2}$

$y = \pm 2$

$x = \pm \sqrt{2}$

For x.y.z

For $x^4 + 4y^4 + 16z^4 + 64 = 32\ xyz$

Either each of x, y, z is (+) ve $\rightarrow$ 1case

Or Two of x, y, z are (–) ve $\rightarrow$ 3case

$\therefore$ 4 case of different (x, y, z)

$\therefore$ 4 possible x + y + z values (as $x \ne y \ne z$)

Q.29 **(A)**

$$M_n = \frac{\sum\limits_{r=1}^{n} 2 + (r-1)4}{n} = 2(n+1) - 2 = 2n$$

Hence $\sum\limits_{n=1}^{n} M_n = 110$

Q.30 **(B)**

Note that $\dfrac{b+c}{2} = bc$

$\Rightarrow$ 4bc – 2b – 2c + 1 = 1

$\Rightarrow$ (2b – 1) (2c – 1) = 1

$\Rightarrow$ b = c = 1

or b = c = 0

Hence a can be 1, 2, 3, 4, –, 9

Q.31 **(D)**

x + y = 1 and x,y > 0

Apply AM $\ge$ HM

$$\frac{x+y}{2} \ge \frac{2}{\dfrac{1}{x} + \dfrac{1}{y}}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} \ge 4$$

JEE MAIN PREVIOUS YEAR'S

Q.1 (1)

$$\text{A.M} \ge \text{G.M} \Rightarrow \frac{a^{a^x} + \dfrac{a}{a^{a^x}}}{2} \ge \left(a^{a^x} \times \frac{a}{a^{a^x}}\right)^{1/2}$$

$$a^{a^x} + \frac{a}{a^{a^x}} \ge 2\sqrt{a}$$

$\therefore$ Minimum value = $2\sqrt{a}$

Q.2 (3)

a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \dfrac{65}{12}$ ......(i)

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{1}{a}\left(\frac{r^3 + r^2 + r + 1}{r^3}\right) = \frac{65}{18} \quad ......(ii)$$

$$\frac{(i)}{(ii)}, a^2r^3 = \frac{18}{12} = \frac{3}{2}$$

$$a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9}r^3 = \frac{3}{2} \quad \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

$$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2}\right)^2 = \frac{3}{2}$$

$2\alpha = 3$

Q.3 (1)

$$S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} ......\infty \quad (i)$$

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + ... (ii)$$

(i) – (ii)

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + ...$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{5}{3^2} + \frac{5}{3^3} + ...$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{\frac{5}{3^2}}{1 - \frac{1}{3}} = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$$

$$S = \frac{13}{6} \times \frac{3}{2} = \frac{13}{4}$$

Q.4 (2)

$$ar + ar^5 = \frac{25}{2} \text{ and } ar^2 \,.\, ar^4 = 25$$

$\Rightarrow ar^3 = 5$

$$\therefore \quad \frac{r + r^5}{r^3} = \frac{5}{2|}$$

$\Rightarrow \quad 2 + 2r^4 = 5r^2$

$\Rightarrow \quad 2r^4 – 5r^2 + 2 = 0$

$\Rightarrow \quad r^2 = 2$ or

$r^2 = \frac{1}{2}$ Reject

Now, $ar^3 + ar^5 + ar^7 = 5 + ar^5 (1 + r^2) = 5 + 5.2 (1 + 2) = 35$

Q.5 (10)

–16, 8, –4, 2

$$p^{th} \text{ term } t_p = -16\left(\frac{-1}{2}\right)^{p-1}$$

$$q^{th} \text{ term } t_q = -16\left(\frac{-1}{2}\right)^{q-1}$$

$$\text{Now } \frac{t_p + t_q}{2} = \frac{5}{4} \,\&\, \sqrt{t_p t_q} = 1$$

$$\Rightarrow 16^2\left(-\frac{1}{2}\right)^{p+q-2} = 1$$

$\Rightarrow 2^8=(–2)^{(p+q–2)}$

$\Rightarrow$ p+q=10

Q.6 **(3)**

GP : 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP : 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

Q.7 **(14)**

$$a2 = \frac{b}{16} \Rightarrow \frac{1}{b} = \frac{1}{16a^2}$$

$$\frac{2}{b} = \frac{1}{a} + 6$$

$$\frac{1}{8a^2} = \frac{1}{a} + 6$$

$$\frac{1}{a^2} - \frac{8}{a} - 48 = 0$$

$$\frac{1}{a} = 12, -4 \quad \Rightarrow \quad a = \frac{1}{12}, -\frac{1}{4}$$

$$a = \frac{1}{12}, \; a > 0$$

$$b = 16a^2 = \frac{1}{9}$$

$\Rightarrow 72 (a + b) = 6 + 8 = 14$

Q.8 **(16)**

$S_n(x) = (2+3+6+11+18+27+.......+\text{n-terms})\log_a x$

Let $S_1 = 2 + 3 + 6 + 11 + 18 + 27 + ... + T_n$

$S_1 = 2 + 3 + 6 + ... + T_n$

____

$T_n = 2 + 1 + 3 + 5 + ...$ + n terms

$T_n = 2 + (n − 1)^2$

$$S_1 = \Sigma T_n = 2n + \frac{(n-1)n(2n-1)}{6}$$

$$\Rightarrow S_n(x) = \left(2n + \frac{n(n-1)(2n-1)}{6}\right)\log_a x$$

$S_{24}(x) = 1093$ (Given)

$$\log_a x \left(48+\frac{23.24.47}{6}\right)=1093$$

$$\log_a x = \frac{1}{4} \qquad .... (1)$$

S_{12} (2x) = 265

S_{12} (2x) = 265

$$\log_a(2x) \left(24+\frac{11.12.23}{6}\right)=265$$

$$\log_a 2x = \frac{1}{2} \qquad .... (2)$$

(2) − (1)

$$\text{loga2x} - \text{logax} = \frac{1}{4}$$

$$\text{loga2} = \frac{1}{4} \Rightarrow a = 16$$

Q.9 (2)

S = (100)(100) + (99)(101) + (98)(102) .....
... (2)(198)+ (1)(199)

$$S=\sum_{x=0}^{99}(100-x)(100+x) = \sum 100^2 - x^2$$

$$=100^3-\frac{99\times100\times199}{6}$$

$\alpha = 3$ $\beta = 1650$

$$\text{slope} = \frac{1650}{3} = 550$$

Q.10 (2)

$$T_n = \frac{1}{(2n+1)^2-1}\frac{1}{(2n+2)2n} = \frac{1}{4(n)\ (n+1)}$$

$$=\frac{n+1(-n)}{4n\ \ n+1}=\frac{1}{4}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

Q.11 (4)

$$S2n = \frac{2n}{2}[2a + (2n-1)d],\ S\ \frac{4n}{2}[2a + (4n-1)d]$$

$$\Rightarrow S2-S1 = \frac{4n}{2}[2a+(4n-1)d]-\frac{2n}{2}[2a+(2n-1)d]$$

= 4an + (4n − 1)2nd − 2na − (2n − 1)dn

= 2na + nd [8n − 2 − 2n + 1]

⇒ 2na + nd [6n − 1] = 1000

$$2a + (6n-1)d = \frac{1000}{n}$$

$$\text{Now, } S6n\ \frac{6n}{2}\left[2a+(6n-1)d\right]$$

$$= 3n.\frac{1000}{n}=3000$$

Q.12 (68)

Let number be a_1, a_2, a_3, ... a_{2n}, b_1, b_2, b_3....bn

$$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$$

$$\Rightarrow \sum a^2 + \sum b^2 = 87n$$

Now, distribution becomes

$a_1 + 1$, $a_2 + 1$, $a_3 + 1$, ...$a2_n + 1$, $b_1 - 1$,
$b_2 - 1$ .....$b_n - 1$

Variance

$$= \frac{\sum(a+1)^2 + \sum(b-1)^2}{3n} - \left(\frac{12n+2n+3n-n}{3n}\right)^2$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2$$

$\Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$

Q.13 (1)

Q.14 (1)

Q.15 (3)

Q.16 (3)

Q.17 (3)

Q.18 (3)

Q.19 (4)

Q.20 (9)

Q.21 (7)

Q.22 (1)

Q.23 (2)

Q.24 (3)

Q.25 (305)

Q.26 (2)

Q.27 (2)

Q.28 (2021)

Q.29 (4)

Q.30 (4)

Q.31 (2)

Q.32 (1)

Q.33 (2)

Q.34 (4)

JEE-ADVANCED PREVIOUS YEAR'S

Q.11 (9)

3 or 9, both 3 and 9 (The common difference of the arithmatic progression can be either 0 or 6, and accordingly the second term can be either 3, or 9 ; thus the answers 3, or 9, or both 3 and 9 are acceptable.)

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6+(5n-1)d]}{\frac{n}{2}[6+(n-1)d]} = \frac{5[(6-d)+5nd]}{[(6-d)+nd]}$$

; d = 6 or d = 0

Now if d = 0 then $a_2 = 3$ else $a_2 = 9$ for single choice more appropriate choice is 9, but in principal, question seems to have an error.

∴ $a_2 = 3 + 6 = 9$

Q.12 (8)

A.M. ≥ G.M.

$$\frac{\frac{1}{a^5}+\frac{1}{a^4}+\frac{1}{a^3}+\frac{1}{a^3}+\frac{1}{a^3}+1+a^8+a^{10}}{8} \geq$$

$$\left(\frac{1}{a^5}.\frac{1}{a^4}.\frac{1}{a^3}.\frac{1}{a^3}.\frac{1}{a^3}.1.a^8.a^{10}\right)^{1/8}$$

$$\Rightarrow \frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^8+a^{10} \geq 8(1)^{1/8}$$

⇒ minimum value of $\frac{1}{a^5}+\frac{1}{a^4}+\frac{3}{a^3}+1+a^3+a^{10}$

= 8, at a = 1

Q.13 (D)

Corresponding A.P.

$\frac{1}{5}$,..................... $\frac{1}{25}$ (20th term)

$$\frac{1}{25} = \frac{1}{5} + 19\,d$$

$$\Rightarrow d = \frac{1}{19}\left(\frac{-4}{25}\right) = -\frac{4}{19\times 25}$$

$a_n < 0$

$$\frac{1}{5} - \frac{4}{19\times 25} \times (n-1) < 0 \qquad \Rightarrow \frac{19\times 5}{4} < n - 1$$

⇒ n > 24 .75

Q.14 (A,D)

$$S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2 = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 +$$

$7^2 + 8^2$ +........

= $(3^2 - 1) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2)$...... =

$$2[\underbrace{4+6+12+14+20+22+......}_{2n\ \text{terms}}]$$

= 2[(4 + 12 + 20 ......) + (6 + 14 + 22 ....)]

n terms

n terms

$$= 2\left[\frac{n}{2}(4\times 2+(n-1)8)\,\frac{n}{2}(2\times 6+(n+1)8)\right] = 2[n(4 + 4n - 4) + n(6 + 4n - 4)]$$

= $2(4n^2 + (4n + 2)n) = 2(8n^2 + 2n) = 4n(4n + 1)$

(A) 1056 = 32 × 33 n = 8

(B) 1088 = 32 × 34

(C) 1120 = 32 × 35

(D) 1332 = 36 × 37 n = 9

Q.15 (5)

Numbers removed are k and k + 1. now $\frac{n(n+1)}{2}$ – k – (k + 1) = 1224

⇒ $n^2 + n - 4k = 2450$ ⇒ $n^2 + n - 2450 = 4k$

⇒ (n + 50)(n – 49) = 4k ⇒ n > 49

Alternative

∴ To satisfy this equation n should be of the form of (4p + 1) or (4p + 2) taking n = 50

⇒ 4k = 100 ⇒ k = 25

∴ k – 20 = 5

Now if we take n = 53 ⇒ k = 103

⇒ n < k

so not possible. Hence n ≥ 53 will not be possible.

Q.16 (4)

Let b = ar, c = ar^2 ⇒ r is Integers. Also $\frac{a + ar + ar^2}{3}$ = ar + 2 ⇒ a + ar^2 = 2ar + 6

⇒ $a(r-1)^2 = 6$ ⇒ r must be 2 and a = 6. Thus

$$\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7} = 4 \text{ Ans.}$$

Q.17 (9)

$$\frac{S_7}{S_{11}} = \frac{6}{11}$$

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11}$$

Given 130 < a + 6d < 140

$$\frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$$

7a + 21d = 6a + 30d ⇒ 130 < 15d < 140

a = 9d Hence d = 9

a = 81

Hence d = 9

Alternative :

Let the AP be a, a + d, a + 2d, ...

where a , d ∈ N

Given $\frac{S_7}{S_{11}} = \frac{6}{11}$ and 130 < a + 6d < 140 ..... (2)

$$\Rightarrow \frac{\frac{7}{2}\{2a+6d\}}{\frac{11}{2}\{2a+10d\}} = \frac{6}{11}$$

$$\Rightarrow \frac{14a+42d}{22a+110d} = \frac{6}{11}$$

⇒ 154a + 462d = 132a + 660d

⇒ 22a = 198d

$\Rightarrow$ a = $\frac{99d}{11}$ = 9d

(2) ⇒ ∴

130 < 9d + 6d < 140

⇒ 8.6 < d < 9.3

∴ d = 9

Q.18 (B)

$\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in A.P

$\Rightarrow b_1, b_2, b_3 ..... b_{101}$ *are in G.P.*

commen deffirence of A.P. is $\log_e 2$, ∴ common ratio of G.P. is 2

$a_1, a_2 ...... a_{101}$ are in A.P

$a_1 = b_1 = A(Let)$

$$Now,\ t = \frac{b_1(2^{51}-1)}{2-1} = b_1(2^{51}-1) = A(2^{51}-1)$$

$$s = \frac{51}{2}\left(2a_1 + (51-1)d\right) = \frac{51}{2}\left(2A + 50d\right)$$

Now, $a_{51} = b_{51} \Rightarrow A + 50d = A\,2^{50} \Rightarrow 50d = A(2^{50}-1)$

$$\therefore\ s = \frac{51}{2}\left[2A + A(2^{50}-1)\right]$$

$$\Rightarrow s = \frac{51}{2}A\left[2^{50}+1\right]$$

$$s = A\left[51.2^{49} + \frac{51}{2}\right] = A\left[4.2^{46} + 47.2^{49} + \frac{51}{2}\right]$$

$$\left[2^{51} - 1 + 41.2^{49} + \frac{51}{2} + 1\right] = A(2^{51}-1) + A\left(47.2^{49} + \frac{53}{2}\right)$$

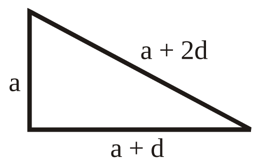
$$\Rightarrow s = t + A\left(47.2^{49} + \frac{53}{2}\right) \Rightarrow s > t$$

$$Now, a_{101} = A + 100d = A + 2A\left(2^{50}-1\right) = A\left(2^{51}-1\right)$$

$$b_{101} = A.2^{100}$$

$Clearly\ b_{101} > a_{101}$

Q.19 (6)



$\frac{1}{2}a\left(a+d\right) = 24$ ⇒

a(a + d) = 48 ....(1)

$a^2 + (a + d)^2 = (a + 2d)^2$ ⇒

$3d^2 + 2ad - a^2 = 0$

(3d – a)(a + d) = 0

⇒ 3d = a (∵ a + d ≠ 0)

⇒ d = 2

a = 6

so smallest side = 6

Q.20 (157.00)

We equate the general terms of three respective A.P.'s as 1 + 3a = 2 + 5b = 3 + 7c

⇒ 3 divides 1 + 2b and 5 divides 1 + 2c

⇒ 1 + 2c = 5, 15, 25 etc.

So, first such terms are possible when 1 + 2c = 15 i.e. c = 7

Hence, first term = a = 52

d = LCM (3,5,7) = 105

⇒ a + d = 157

Q.21 (8.00)

$$\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3} \geq \left[3^{(y_1+y_2+y_3)}\right]^{\frac{1}{3}}$$

$$\Rightarrow 3^{y_1} + 3^{y_2} + 3^{y_3} \geq 3^4$$

$$\Rightarrow \log_3\left(3^{y_1} + 3^{y_2} + 3^{y_3}\right) \geq 4$$

⇒ m = 4

Also, $\frac{x_1 + x_2 + x_3}{3} \geq \sqrt[3]{x_1 x_2 x_3}$

$\Rightarrow x_1 x_2 x_3 \leq 27$

$\Rightarrow \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

⇒ M = 3

Thus, $\log_2(m^3)+\log_3(M^2) = 6+2$
$= 8$

Q.22 (1.00)

Given $2(a_1+a_2+....a_n) = b_1+b_2+....b_n$

$$\Rightarrow 2\times\frac{n}{2}\left(2c+(n-2)2\right) = c\left(\frac{2^n-1}{2-1}\right)$$

$\Rightarrow 2n^2-2n=c(2^n-1-2n)$

$$\Rightarrow c=\frac{2n^2-2n}{2^n-1-2n}\in\mathbb{N}$$

So, $2n^2-2n \ge 2^n-1-2n$

$\Rightarrow 2n^2+1\ge 2^n \quad \Rightarrow \ n<7$

$\Rightarrow$ n can be 1, 2, 3....,

Checking c against these values of n

we get c = 12 (when n = 3)

Hence number of such c = 1

Q.23 (D)

$$S_n = 1+\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{n-1}}$$

$$=2\left(1-\frac{1}{2^n}\right)=2-\frac{1}{2^{n-1}}$$

Centre of C_n is $\left(2-\frac{1}{2^{n-2}},0\right)$

and radius of C_n is $\frac{1}{2^{n-1}}$

when $r=\frac{1025}{S13}<2$

C_n will lie inside m

when $2-\frac{1}{2^{n-2}}+\frac{1}{2^{n-1}}<\frac{1025}{S13}$

$\Rightarrow k = 10$

Also $\ell = 5$

$3k + 2\ell = 30 + 10 = 40$

Ans. (D)

Q.24 (B)

Centre of D_n is (S_{n-1}, S_{n-1})

$r = \frac{1}{2^{n-1}}$

D_n will lie inside

when $\sqrt{2}(S_{n-1}) < \frac{2^{199}-1}{2^{198}}\sqrt{2}$

$$\Rightarrow \frac{\sqrt{2}}{2^{n-2}} > \frac{\sqrt{2}}{2^{198}}+\frac{1}{2^{n-1}}$$

$\Rightarrow n = 199$

Permutations and Combinations

EXERCISES

ELEMENTRY

Q.1 (3)

1 digit : 3 ways (3, 5 or 7)

2 digits : 3 × 3 = 9 (last digit has three options, which leaves 3 options for the first digit)

3 digits : 3 × 2 × 3 = 18 (3 options for last digit, then 3 for first digit and 2 for second)

4 digits : 3 × 2 × 1 × 3 = 18 (3 options for last digit, 3! for remaining digits)

⇒ 3 + 9 + 18 + 18 = 48.

Q.2 (3)

Q.3 (3)

Since ${}^nC_2 - n = 35 \Rightarrow \frac{n!}{2!\,(n-2)!} - n = 35$

$\Rightarrow n(n-1) - 2n = 70 \Rightarrow n^2 - 3n = 70$

$\Rightarrow n^2 - 3n - 70 = 0 \Rightarrow (n+7)\,(n-10) = 0 \Rightarrow n = 10$

Q.4 (3)

Since the 5 boys can sit in 5 ! ways. In this case there are 6 places are vacant in which the girls can sit in ${}^6P_3$ ways. Therefore required number of ways are ${}^6P_3 \times 5\,!$

Q.5 (1)

Required number of ways

$= {}^{15}C_1 \times {}^8C_1 = 15 \times 8$

Q.6 (1)

${}^{15}C_{3r} = {}^{15}C_{r+3} \Rightarrow {}^{15}C_{15-3r} = {}^{15}C_{r+3}$

$\Rightarrow 15 - 3r = r + 3 \Rightarrow r = 3$

Q.7 (2)

${}^nC_2 = 153 \Rightarrow \frac{n(n-1)}{2} = 153 \Rightarrow n = 18$

Q.8 (2)

Since 5 are always to be excluded and 6 always to be included, therefore 5 players to be chosen from 14. Hence required number of ways are ${}^{14}C_5 = 2002$.

Q.9 (2)

Required number of ways $= {}^4C_2 \times {}^3C_2 = 18$

Q.10 (4)

The required number of points

$= {}^8C_2 \times 1 + {}^4C_2 \times 2 + \left({}^8C_1 \times {}^4C_1\right) \times 2$

$= 28 + 12 + 32 \times 2 = 104$

Q.11 (2)

Clearly, ${}^nC_3 = T_n$.

So, ${}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow ({}^nC_3 + {}^nC_2) - {}^nC_3 = 21$

$\therefore\ {}^nC_2 = 21$ or $n(n-1) = 42 = 7.6\ \therefore\ n = 7$

Q.12 (1)

(2) Required number of ways $= {}^{30}C_3 - {}^{15}C_3 = 3605$

Q.13 (1)

Number of triangles $= {}^{10}C_3 - {}^6C_3 = 120 - 20 = 100$.

Q.14 (3)

Given, total number of points = *n* and number of collinear points = *p*. We know that one line has two end points. Therefore total number of lines = ${}^nC_2$. Since *p* points are collinear, therefore total number of lines drawn from collinear points = ${}^pC_2$. We also know that, corresponding to the line of collinearity, one will also be added.

Therefore number of lines = ${}^nC_2 - {}^pC_2 + 1$.

Q.15 (3)

A gets 2, B gets 8; $\frac{10\,!}{2\,!\,8\,!} = 45$

A gets 8, B gets 2; $\frac{10\,!}{8\,!\,2\,!} = 45$

$45 + 45 = 90$

Q.16 (1)

Required number of ways

$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}$

$= \frac{52\,!}{39\,! \times 13\,!} \times \frac{39\,!}{26! \times 13\,!} \times \frac{26\,!}{13\,! \times 13\,!} \times \frac{13\,!}{13\,!} = \frac{52\,!}{(13\,!)^4}$

Q.17 (3)

The number of ways can be given as follows

2 bowlers and 9 other players $= {}^4C_2 \times {}^9C_9$

3 bowlers and 8 other players $= {}^4C_3 \times {}^9C_8$

4 bowlers and 7 other players $= {}^4C_4 \times {}^9C_7$

Hence required number of ways

$= 6 \times 1 + 4 \times 9 + 1 \times 36 = 78$

Q.18 (1)

The selection can be made in ${}^5C_3 \times {}^{22}C_9$

{Since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

Q.19 (4)

A garland can be made from 10 flowers in $\frac{1}{2}(9!)$ ways.

$\{\because$ n flowers' garland can be made in $\frac{1}{2}(n-1)!$ ways$\}$

Q.20 (2)

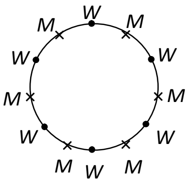
Since total number of ways in which boys can occupy any place is $(5-1)! = 4!$ and the 5 girls can be sit accordingly in $5!$ ways.

Hence required number of ways are $4! \times 5!$

Q.21 (1)

Fix up a male and the remaining 4 male can be seated in 4! ways. Now no two female are to sit together and as such the 2 female are to be arranged in five empty seats between two consecutive male and number of arrangement will be ${}^5P_2$. Hence by fundamental theorem the total number of ways is = $4! \times {}^5P_2$ = 24 × 20 = 480 ways.

Q.22



No. of ways in which 6 men can be arranged at a round table = (6 – 1)!

Now women can be arranged in 6! ways.

Total Number of ways = 6! × 5!

Q.23 (2)

Required number of ways

$$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^2 - 1 = 63$$

Q.24 Required number of ways = $2^{10} - 1$

(Since the case that no friend be invited *i.e.,* ${}^{10}C_0$ is excluded).

Q.25 (1)

Required number of ways

$$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + ... + {}^{12}C_{12} = 2^{12} - 1$$

$$= 4096 - 1 = 4095$$

Q.26 (4)

Word 'MATHEMATICS' has 2*M*, 2*T*, 2*A*, *H*, *E*, *I*, *C*, *S*. Therefore 4 letters can be chosen in the following ways.

Case I : 2 alike of one kind and 2 alike of second kind i.e., ${}^3C_2 \Rightarrow$ No. of words $= {}^3C_2 \frac{4!}{2!2!} = 18$

Case II : 2 alike of one kind and 2 different *i.e.,* ${}^3C_1 \times {}^7C_2 \Rightarrow$

No.of words $= {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$

Case III : All are different

i.e. ${}^8C_4 \Rightarrow$ No. of words ${}^8C_4 \times 4! = 1680$

Hence total number of words are 2454.

Q.27 (1)

Since, 38808 = 8 × 4851

$$= 8 \times 9 \times 539 = 8 \times 9 \times 7 \times 7 \times 11 = 2^3 \times 3^2 \times 7^2 \times 11$$

So, number of divisors

= (3 + 1) (2 + 1) (2 + 1) (1 + 1) = 72.

This includes two divisors 1 and 38808. Hence, the required number of divisors = 72 – 2 = 70.

Q.28 (2)

We have, 30 = 2 × 3 × 5. So, 2 can be assigned to either *a* or *b* or *c i.e.* 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the no. of solutions are 3 × 3 × 3 = 27.

Q.29 (2)

Number of ways to distribute 15 balls among 4 children such that no child will get more than 5 balls = Number of ways to distribute 5 balls among 4 children

Number of ways = ${}^8C_3 = 56$

Q.30 (1)

Since the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$

Therefore the required number $= {}^{3+6-1}C_6 = 28$

Q.31 (1)

x = 5a + 3

y = 5b + 3

z = 5c + 3

w = 5d + 3

x + y + z + w = 5(a + b + c + d) + 12 = 62 = {a + b + c + d = 10}

so ${}^{10+4-1}C_{4-1} = {}^{13}C_3$

Q.32 (D)

C = A × B ⇒ n(C) = 3 × 4 = 12

For each element of C there are 3 choices either P or Q or neither P nor Q

Number of ways = 3^{12}]

Q.33 (2)

Three letters can be posted in 4 letter boxes in 4^3 = 64 ways but it consists the 4 ways that all letters may be posted in same box. Hence required ways = 60.

Q.34 (3)

Let E(n) denote the exponent of 3 in n. The greatest integer less than 100 divisible by 3 is 99.

We have $E(100\,!) = E(1\,.\,2\,.\,3\,.\,4....99\,.\,100)$

$= E(3\,.\,6\,.\,9\,....99) = E[(3\,.\,1)(3\,.\,2)(3\,.\,3)........(3\,.\,33)]$

$= 33 + E(1\,.\,2\,.\,3......33)$

Now $E(1\,.\,2\,.\,3......33) = E(3\,.\,6\,.\,9....33)$

$= E[(3\,.\,1)(3\,.\,2)(3\,.\,3)........(3\,.\,11)]$

$= 11 + E(1\,.\,2\,.\,3\,.....11)$

and

$E(1\,.\,2\,.\,3\,....11) = E(3\,.\,6\,.\,9) = E[(3\,.\,1)(3\,.\,2)(3\,.\,3)]$

$3 + E(1\,.\,2\,.\,3) = 3 + 1 = 4$

Thus $E(100\,!) = 33 + 11 + 4 = 48$.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 **(2)**

As per the given condition, digit 1 should occur at alternate places of the number and at the remaining 5 places either 2, 3, 5 or 7 should appear. Now when the number starts with 1, number of numbers = 4^5 and when the number starts with either 2, 3, 5 or 7, number of numbers = 4^5

So, total number = 2×4^5 = 2048 **Ans.**

Q.2 **(1)**

1 + 2 + 3 + ... + 9 = 45 = 0 + 1 + 2 + 3 + ... + 9

All 9 digit such numbers = 9 !

All 10 digit such numbers when '0' included = 10 ! – 9 !

So, total = 9 ! + (10 ! – 9 !) = (10) ! **Ans.**

Q.3 **(4)**

1 . ② . 3 . ④ . 5 . ⑥ . 7

T					T

$^3C_2 \cdot 2! \cdot {}^5C_4 \cdot 4! = 6 \times 120 = 720$]

Q.4 **(3)**

Word QUEUE

E → 2, Q , U - 2

E				

= 18

$\frac{4!}{2!}$

Q	E			

= 3

$\frac{3!}{2!}$

Q	U	E	E	U

= 1

Q	U	E	U	E

= 1

17th rank

Q.5 **(2)**

$$^{2002}C_{1001} = \frac{(2002)!}{(1001)!(1001)!}$$

no. of zeros in (2002)! are

400 + 80 + 16 + 3 = 499

no. of zeroes in $(1001\,!)^2$ = 2(200 + 40 + 8 + 1) = 498

Hence no. of zeroes is $\frac{(2002)!}{(1001!)^2}$ = 1

Q.6 **(3)**

Total number of signals can be made from 3 flags each of different colour by hoisting 1 or 2 or 3 above.

i.e. $^3p_1 + {}^3p_2 + {}^3p_3 = 3 + 6 + 6 = 15$

Q.7 **(4)**

Total number of possible arrangements is

$^4p_2 \times {}^6p_3$.

Q.8 **(4)**

First we have to find all the arrangements of the word ‘GENIUS’ is

6 ! = 720

number of arrangement which in either started with G ends with S is

(5! + 5! – 4!) = (120 + 120 – 24)= 216

Hence total number of arrangement which is neither started with G nor ends with S is.

(720 – 216) = 504

Q.9 **(1)**

Total no. of arrangement if all the girls do not sit side by side is = [all arrangement – girls seat side by side]

= 8! – (6! × 3!) = 6! (56 –6) = 6! × 50 = 720 × 50 = 36000

Q.10 **(1)**

Number of words which have at least one letter repeated = total words – number of words which have no letter repeated = $10^5 - 10 \times 9 \times 8 \times 7 \times 6$ = 69760

Q.11 **(4)**

First we select 3 speaker out of 10 speaker and put in any way and rest are no restriction i.e. total

number of ways = $^{10}C_3 \,.\, 7!.2\,! = \frac{10!}{3}$

Q.12 **(2)**

upperdeck - 13 seats → 8 in upper deck.

lowerdeck - 7 seats → 5 in lower deck

Remains passengers = 7

Now Remains 5 seats in upper deck and 2 seats in lower deck for upper deck number of ways = $^7C_5$

for lower deck number of ways = $^2C_2$

So total number of ways = $^7C_5 \times {}^2C_2 = \frac{7.6}{2} = 21$

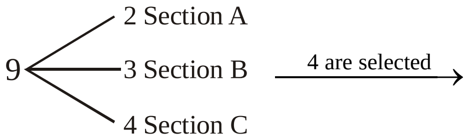
Q.13 **(4)**

Peaches 5 p_1 , p_2 , p_3, p_4, p_5

Apples 3 a_1 , a_2 , a_3

Hence number of ways = $^3C_1 \times {}^5C_3 = 30$ **Ans.**

Q.14 **(3)**



required number of ways

A	B	C
2	1	1
1	1	2
1	2	1

Hence $^2C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^2C_1 \cdot {}^3C_2 \cdot {}^4C_1 + {}^2C_2 \cdot {}^3C_1 \cdot {}^4C_1 = 36 + 24 + 12 = 72$ **Ans.**

Alternatively:

$$^9C_4 - \underbrace{\left[{}^7C_4 + {}^6C_4 + {}^5C_4 + {}^4C_4\right]}_{\text{think!}} = 126 - 56$$

= 72 **Ans.**

Q.15 **(2)**

A E L R Y

A _ _ _ _ = 4! = 24

E A L _ _ = 2! = 02

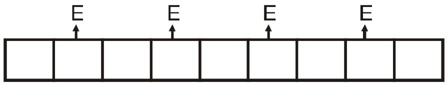
E A R L Y = 1 = 01

= 27.

The number of words before EARLY = 27 – 1 = 26.]

Q.16 **(4)**

Even place



There are four even places and four odd digit number so total number of filling is $\frac{4!}{2!.2!}$ rest are also occupy is $\frac{5!}{3!.2!}$ ways

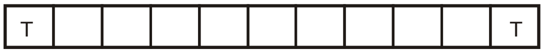
Hence total number of ways = $\frac{4!}{2!.2!} \times \frac{5!}{3!.2!} = 60$

Q.17 **(3)**

They can sit in groups of either 5 and 3 or 4 and 4

required number = $\frac{8!}{5! \times 3!} \times 1 + \frac{8! \times 2!}{4! \times 4! \times 2!} = 126$

Q.18 **(2)**



Total arrangement is $\frac{9!}{2!.2!} = 90720$

Q.19 **(3)**

NINETEEN

⇒ N → 3 : I, T

E → 3

First we arrange the word of N, N, N, I and T

then the number of ways = $\frac{5!}{3!}$.

Now total 6 number of place which are arrange E is $^6C_3$

Hence total number of ways = $\frac{5!}{3!} \cdot {}^6C_3$

Q.20 **(1)**

Total number of ways of arranging 2 identical white balls.

3 identical red balls and 4 green balls of different shades = $\frac{9!}{2!3!} = 6.7!$

Number of ways when balls of same colour are together = 3! × 4! = 6.4!

∴ Number of ways of arranging the balls when atleast one ball is separated from the balls of the same colour = 6.7! – 6.4! = 6(7! – 4!)

Q.21 **(3)**

only 7, 8 and 9 can be used

Aliter: 9, 9, 9, 9, 9, 9, 7 $\rightarrow \frac{7!}{6!} = 7$

9, 9, 9, 9, 9, 8, 8 $\rightarrow \frac{7!}{2!5!} = 21$

Total = 28 **Ans.**

Q.22 **(3)**

Total number of ways is

$\frac{6! \times 3!}{2!} = 720 \times 3 = 2160$

Q.23 **(1)**

First we select 5 beads from 8 different beads to $^8C_5$

Now total number of arrangement is

$$^8C_5 \times \frac{4!}{2!} = 672$$

Q.24 **(4)**

Total number of proper divisors is

$(p + 1)(q + 1)(r + 1)(s + 1) - 2$

(Number and 1 are not proper divisor)

Q.25 **(1)**

$N = 2^{\alpha}.3^{\beta}5^{\gamma} = 2^3.3^2.5$

$(\alpha + 1)(\beta + 1)(\gamma + 1) = 4.3.2$

$N = 360 = 2^3 . 3^2 .5$

$$\frac{4.3.2}{2} = 12$$

Q.26 **(1)**

Here $21600 = 2^5. 3^3. 5^2 \Rightarrow (2 \times 5) \times 2^4 \times 3^3 \times 5^1$

Now numbers which are divisible by 10 $= (4 + 1)(3 + 1)(1 + 1) = 40$

$(2 \times 3 \times 5) \times (2^4 \times 3^2 \times 5^1)$ now numbers which are divisible by both 10 and 15

$= (4 +1)(2 +1)(1+1) = 30$

So the numbers which are divisible by only

$40 - 30 = 10$

Q.27 **(1)**

Coefficient x^{10} in $(x + x^2 .....x^5)^6$ = coefficient of x^4 in $(x^0 + x^1 ....x^4)^6 = {}^{6+4-1}C_4 = {}^9C_4 = 126$

Alternatively: Give one apple to each child and then for rest 4 apples $= {}^{4+6-1}C_{6-1} = 126$

Q.28 **(2)**

3 A + 2 O. A. = 3 . 2 = 6 ; 3 A + 2 diff = 3 ;

2 A + 2 O. A. + 1 D = 3 ⇒ 12

Q.29 **(3)**

$(x + y + z)^n \rightarrow$ use beggar]

Q.30 **(3)**

If 1 be unit digit then total no. of number is 3! = 6

Similarly so on if 3, 5, or 7 be unit digit number then total no. of no. is 3! = 6

Hence sum of all unit digit no. is = 6× (1+3+5+7) = 6× 16 = 96

Hence total sum is $= 96 \times 10^3 + 96 \times 10^2 + 96 \times 10^1 + 96 \times 10^0$

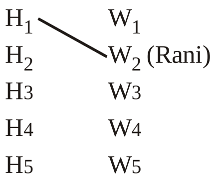
$= 96000 + 9600 + 960 + 96 = 106656 = 16 \times 1111 \times 3!$

Q.31 **(1)**

$(1 + 10 + 10^2 + 10^3) \times 4^3 \times (6 + 7 + 8 + 9) = (1111) \times 64 \times 30 = 2133120$

Q.32 **(2)**

Naresh



No husband dances with his own wife.

Case-I: H_1 dance with W_2 and H_2 dance with W_1 then required case

$$= 3 : \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) = 3 - 1 = 2$$

Case-II: When H_1 dance with W_2 and H_2 will not dance with W_1.

$$= 4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 12 - 4 + 1 = 9$$

Total = 11 **Ans.**

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (**A**)

There are 900 three digit numbers and there are five odd digits. Thus, there are $5^3 = 125$ three digit numbers comprised of only odd digits. The other 900 – 125 = 775 three digit numbers must contain at least one even digit.

Q.2 (**C**)

We have N =

a	b	c	d

First place a can be filled in 2 ways i.e. 4, 5 $(4000 \le N < 6000)$

For b and c, total possibilities are '6' $(3 \le b < c \le 6)$

i.e. 34, 35, 36, 45, 46, 56

Last place d can be filled in 2 ways i.e. 0, 5 (N is a multiple of 5)

Hence total numbers = 2 × 6 × 2 = 24 **Ans.**

Q.3 (**C**)

Out of 8 integers 1,.....8 the pairs of twin primes are (3, 5), (5, 3), (5, 7), and (7, 5). We consider the following 3 cases.

Hence 4 × 6! = 2880.

Q.4 **(B)**

$\beta - \alpha$

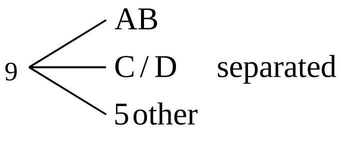
$= y_1\, y_2\, y_3$

$- x_1\, x_2\, x_3$

____

Number of pairs $= (10 + 9 + 8 + ... + 1)^2 . (9 + 8 + ... + 1) = (55)^2 .45$

Q.5 (**A**)



AB included ${}^7C_3 - {}^5C_1 = 30$ (${}^7C_3$ denotes any 3 from (CD and 5 others – no. of ways when CD)

AB excluded ${}^7C_5 - {}^5C_3 = 11$ is taken and one 3 from remaining five)

—— 41

Q.6 (**B**)

First we find 3 ball from 9 ball

$^9C_3 = 84$

Now number of ways if any one black ball not selected is $^6C_3 = 20$

Here required no is 84 - 20 = 64

Q.7 (**A**)

First fill 3 places by 1,2,3 in $^5P_3$ ways and then remaining one in 7 x 7 ways so total no. of ways

$= {}^5P_3 \times 7 \times 7 = 2940$

Q.8 (**B**)

Number of bowlers = 4

Number of wicketkeeper = 2

Let total number required selection

$^4C_3 . {}^2C_1 . {}^{10}C_7 + {}^4C_4 . {}^2C_1 . {}^{10}C_6 + {}^4C_3 . {}^2C_2 . {}^{10}C_6 + {}^4C_4 . {}^2C_2 . {}^{10}C_5$

960 + 420 + 840 + 252 = 2472

Q.9 (**A**)

$3\,({}^4C_2 . {}^4C_2 . {}^4C_1) + 3\,({}^4C_1 . {}^4C_1 . {}^4C_3) = 432 + 194 = 624$]

Alternative :

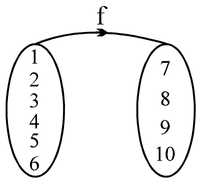
Total – [no of ways in which he does not select any question from any one section]

$^{12}C_5 - 3 \cdot {}^8C_5$; Note that $^4C_1 . {}^4C_1 . {}^4C_1 . {}^9C_2$ is wrong think !]

Q.10 (**B**)

Giving any random configuration for f (1) to f (5), number of functions = 4^5 ways (each element from 1 to 5 can be mapped in 4 ways)

Now the sum $\sum_{i=1}^{5} f(i)$ is either odd or even



if it is odd then we have only two choices for f (6) i.e. the element 8 or 10.

(as odd + even = odd).

If the sum is even then also we have only choices for f (6) i.e. 7 or 9.

hence the total functions = $4^5 \cdot 2 = 2^{11}$ **Ans.**]

Q.11 (**B**)

$$\times\ \times\ \boxed{AB}\ \times\ \times\ \times\ \times$$

C D

$\boxed{AB}$ and 6 other is 7! but A and B can be arranged in 2! ways

$\therefore$ Total ways = $7! \cdot 2!$

when C is behind D

$\therefore$ required number of ways = $\frac{7! \cdot 2!}{2!} = 5040$ ways **Ans.**

Q.12 (**B**)

$$\left(\frac{4!}{2!}\right)\left(\begin{matrix}2\text{ ways}\\ \text{for fifth place}\end{matrix}\right)\left(\begin{matrix}10\text{ ways}\\ 6^{th}\text{ place}\end{matrix}\right)\left(\begin{matrix}1\text{ way}\\ 7^{th}\text{ place}\end{matrix}\right)$$

$$\underbrace{\times\times\times\times}_{1\;2\;3\;3}\times\times\underbrace{\times}_{7^{th}}$$

$x_7 = 9 - x_6$

x_6 can take 0 to 9

= 240]

Q.13 (**B**)

Total no. of different tickets is 13 + 12 + 11 + 10 + ...+ 1 = 91

Hence required no. = $^{91}C_{75} = {}^{91}C_{88}$

Q.14 (C)

$^{11}C_3 + {}^{11}C_4 + \ldots\ldots + {}^{11}C_{11} = 2^{11} - {}^{11}C_0 - {}^{11}C_1 - {}^{11}C_2 = 1981$

Q.15 (**C**)

If 'n' straight line intersect each other then total number of intersection point is $^nC_2 = \frac{n(n-1)}{2}$

Now, from these $^nC_2$ points we can make $^{\frac{n(n-1)}{2}}C_2$ lines. (total old + new lines) and number of old lines are $^{n-1}C_2 \times n$

So fresh lines are $^{\frac{n(n-1)}{2}}C_2 - {}^{n-1}C_2 \times n = \frac{1}{8}\, n\,(n-1)\,(n-2)\,(n-3)$

Q.16 (**A**)

We know that in odd sides polygon no two or more then two diagonals are parallel, so if we take any 4 vertices, we get one point of intersection of diagonals,

Hence required no of points will be $^{2009}C_4$.

Q.17 (**A**)

India wins exactly in 5 matches ⇒ looses in none ⇒ $^5C_0$ ways

India wins exactly in 6 matches ⇒ wins the 6th and looses anyone in the 1st five

⇒ $^5C_1$ ways and so on . ⇒ $^5C_0 + {}^5C_1 + {}^6C_2 + {}^7C_3 + {}^8C_4 = 126$

Alternatively :

Total number of ways in which Series can be won by India or Pakistan = $^{10}C_5$

⇒ required number of ways = $\frac{{}^{10}C_5}{2} = 126$

Q.18 (**B**)

Number of trips which exceeds ≡ when one kid is never included ⇒ $^{25}C_5 - {}^{24}C_4 = {}^{24}C_5$]

Q.19 **(D)**

A A A ...., B B B ...., C C C ....

X X X X X; 3^5 – [No. of ways when exactly one colour counter is not there + when exactly two colour counters are not there]

$= 3^5 - [{}^3C_1\,(2^5 - 2) + {}^3C_2]$ ....(1)

alternately

A B C X X

we can choose both alike counters for two crosses or we can choose both different.

Accordingly

$${}^3C_1 \cdot \frac{5\,!}{3\,!} + {}^3C_2 \cdot \frac{5\,!}{2\,!\;2\,!}$$ ....(2)

(1) and (2) gives 150 ⇒ D

Q.20 **(B)**

Method - 1

Case -1

Delegation consists of 5 members

(a) 1 Boy + 4 Girls $\quad {}^2C_1 \times {}^6C_4 = 30$

(b) 2 Boys + 3 Girls $\quad {}^2C_2 \times {}^6C_3 = \underline{20}$

50

Case -2

Delegation consists of 6 members

(a) 1 Boy + 5 Girls $\quad {}^2C_1 \times {}^6C_5 = 12$

(b) 2 Boys + 4 Girls $\quad {}^2C_2 \times {}^6C_4 = 15$

______

77 ways ______

Method - 2

Case -1

Deligation consists of 5 members.

Total number of ways of selecting 5 member team – number of ways when no boy is selected

$= {}^8C_5 - {}^6C_5 = 50$

Case -2

Deligation consists of 6 members

∴ Total - no boy is selected $= {}^8C_6 - {}^6C_6 = 27$

Total = 50 + 27 = 77 **Ans.**

Q.21 **(C)**

Here first three children receive 2 each and younger receives 3 toys

then total number of distribution is

$${}^9C_2 \,.\, {}^7C_2 \,.\, {}^5C_2 \,.\, {}^3C_2 \times \frac{3!}{3!}$$

$$= \frac{9!}{2!.7!}.\frac{7!}{2!.5!}.\frac{5!}{2!.3!}.\frac{3!}{3!.0!} = \frac{9!}{3!.(2!)^3}$$

Q.22 **(C)**

$$L = \text{Grouping} : \frac{(p+q)!}{p!\,.\,q!} \qquad M : \left(\frac{(p+q)!}{p!\,.\,q!}\right).2!$$

$$N : \frac{(p+q)!}{p!\,q!}$$

Q.23 **(B)**

Total card = 52

4 ace + 4 king + 4 Queen + 4 Jack = 16 card

Remaining card = 52 – 16 = 36

Distribute equally between four players

$$\text{ways} = \frac{\lfloor\underline{36}}{\lfloor\underline{9}\lfloor\underline{9}\lfloor\underline{9}\lfloor\underline{9}}$$

ways for distributing

ace, king, queen, jack of same suit = $\lfloor\underline{4}$

$$\text{Total ways} = \frac{\lfloor\underline{36}\lfloor\underline{4}}{(\lfloor\underline{9})^4}$$

Q.24 **(A)**

First we select n grand children from 2n grand children is ${}^{2n}C_n$

Now arrangement of both group is n! × n!

Now Rest all (m + 1) place where we occupy the grandfather and m sons but grandfather refuse the sit to either side of grand children so the out of m – 1 seat one seat can be selected

Now required number of sitting in ${}^{2n}C_n \times n\,! \times n\,! \times {}^{(m-1)}C_1 \,.\, m\,!$

$$= \frac{12n}{n!\times n!} \times n\,! \times n\,! \times {}^{(m-1)}C_1 \,.\, m\,! = 2n\,!\,.\,m\,!\,.\,(m-1)$$

Q.25 **(C)**

There are 9 married couple so first we select 2 man out of 9 and then we select 2 women out of rest 7 then, we arranged them, so required no. is ${}^9C_2 \times {}^7C_2 \times 2! = 36 \times 21 \times 2 = 1512$

Q.26 **(D)**

A	B	A	B	A	B

= total number of required possible is

B	A	B	A	B	A

$${}^{12}C_6 \times 6\,! \times 6\,! \times 2\,! = \frac{12!}{6!\times 6!} \times 6\,! \times 6\,! \times 2\,!$$

= 2 × 12 !

Q.27 **(A)**

First we select 3 length from the given 6 length so the no. of ways = ${}^6c_3$

But these some pair i.e. (2, 3, 7), (2, 3, 6), (2, 3, 5) (2, 4, 6), (2, 4, 7), (2, 5, 7), (3, 4, 7) are not form a triangle so that total no. of ways is ${}^6C_3$ – 7 ways

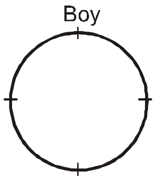
Q.28 **C**

First be find all 4 particulars flowers are together then the total number of ways is

$\frac{4!\times 4!}{2}$ = ... 288

Q.29 **A**

First we arrange all the boy so no. of ways of all the boy can stand is 3! now we arrange all the girl in 4 ! ways so total no. of ways is



= 4! × 3!

Q.30 (**B**)

Indians - 2
Americans - 3
Italians - 3
Frenchmen - 4

total number arranging in row of same nationality are together

= 3 ! × 2! × 3! × 3! × 4 !

= 3 ! × 2! × 3! × 3! × 4 !   = 2 . $(3!)^3$. 4!

Q.31 (**A**)

Words starting with A

$\frac{10!}{2!2!} = \frac{10.9.8.7.6.5.4.3.2.1}{2.2}$ = 907200.

Words starting with E

E : $\frac{10!}{2!2!2!} = \frac{907200}{2}$ = 453600.

Words starting with I

$\frac{10!}{2!2!}$ = 907200.

Hence Total word starting with M = 907200 + 453600 + 907200 = 2268000.

Q.32 (**A**)

Word is PROPOSITION

Here P's = 2, O's = 3, I's = 2 and T, R, N, S = 1

We have to made 5 letters words

Case I : When All 5 are different = ${}^7p_5 = \frac{7!}{2!}$ = 2520

Case II : When 2 alike ,3 different = ${}^3C_1 \times {}^6C_3 \times \frac{5!}{2!}$ = 3600

Case III : When 2 alike , 2 alike, 1 different = ${}^3C_2 \times {}^5C_1 \times \frac{5!}{2!2!}$ = 450

Case IV : When 3 alike, 2 different = ${}^1C_1 \times {}^6C_2 \times \frac{5!}{3!}$ = 300

Case V : When 3 alike , 2 alike = ${}^1C_1 \times {}^2C_1 \times \frac{5!}{3!2!}$ = 20

So total required number = 6890 **Ans.**

Q.33 (**B**)

We have to select At least one fruit from 5 mangoes, 4 Apples , 3 Bananas and 3 different fruit is .

$(5 + 1)(4 + 1)(3 + 1)2^3 - 1 = 6 \times 5 \times 4 \times 8 - 1$ = 960 – 1 = 959 **Ans.**

exactly one fruit selection from 5 mangoes, 4 Apples , 3 Bananas and 3 different fruit is.

$(1 + 1 + 1 + {}^3C_1)$ = 6 ways

So number of selection at least 2 fruit from 5 mangoes, 4 Apples , 3 Bananas and 3 different fruit

= 959 – 6 = 953

Q.34 **C**

G G G G G G G / B B B B B B B B

one gap out of nine can be taken in ${}^9C_1$ ways

now green remaining 5

gaps remaining 8

5 gaps for remaining 5 green can be selected in ${}^8C_5$

Hence Total ways ${}^9C_1 \cdot {}^8C_5 = 9 \cdot \frac{8\cdot7\cdot6}{1\cdot2\cdot3} = 72 \cdot 7$ = 504 Ans.

Alternatively:

${}^9C_6 \times 6$ = 504 Ans. (think ! how)

(6 bottle in 6 gaps and the remaining in any one of these 6 gaps)

Q.35 **(D)**

Total n-digit numbers using 1, 2 or 3 = 3^n

total n-digit numbers using any two digits out of 1, 2 or 3 = ${}^3C_2 \times 2^n - 6 = 3 \times 2^n - 6$

total n-digit numbers using only one digit of 1, 2 or 3 = 3

∴ the numbers containing all three of the digits 1, 2 and 3 at least once = $3^n - (3 \times 2^n - 6) - 3 = 3^n - 3 \cdot 2^n + 3$

Q.36 (**B**)

2	27720
2	13860
5	6930
2	1386
3	693
3	231
7	77
	11

$2^3\ 3^2 . 5. 11$
Hence number of co-prime factor
$2^{5-1} = 2^4 = 16$

Q.37 **(B)**

$$\boxed{\ }\boxed{\ }\boxed{\ }\boxed{0}\boxed{\ }\boxed{\ }\boxed{\ } = \frac{6\,!}{2\,!\ 3\,!\ 1\,!} = 60$$

$$\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{5} = \frac{6\,!}{2\,!\ 3\,!\ 1\,!} - \frac{5\,!}{2\,!\ 3\,!} = 50$$

$\Rightarrow 60 + 50 = 110$

Q.38 **(C)**
Total number of positive integral solution of
$x_1 . x_2 . x_3 = 30 = 2 \times 3 \times 5$ is
$3 \times 3 \times 3 = 27$

Q.39 **(A)**
For P → If same species are different
Total number of arragements is ${}^{n}P_2 . (m + n - 2)\,!$
For Q → If same species are alike then number of arrangement is $\dfrac{(m+n-2)!}{m!.(n-2)!}$

Hence $\dfrac{P}{Q} = {}^{n}P_2 . m! . (n-2)! = {}^{n}P_2 . {}^{m}P_m . (n-2)!$

Q.40 **(D)**
Here we should go (n –1) steps to east and (m –1) steps to south so total steps which we have to go are (m + n – 2) ways.
Hence total no. of ways = ${}^{m+n-2}C_{m-1} . {}^{n-1}C_{n-1}$

$$= \frac{(m+n-2)\,!}{(m-1)\,!\,.\,(n-1)\,!}$$

Q.41 **(A)**
Using Beggar's Method
Total no. of ways of choosing 6 chocolates out of 8 different brand is = ${}^{8+6-1}C_6 = {}^{13}C_6$

Q.42 **(C)**
Using Beggar's Method
Total no. of ways = ${}^{15+3-1}C_{15} \times {}^{10+3-1}C_{10} = {}^{17}C_{15} \times {}^{12}C_{10} = \dfrac{17\times16}{2}\times\dfrac{12\times11}{2} = 8976$

Q.43 **(B)**
$P_n = {}^{n-2}C_3$; $P_{n+1} = {}^{n-1}C_3$; Hence ${}^{n-1}C_3 - {}^{n-2}C_3 = 15$

Q.44 **(A)**
Ordered pair = total – (A ∪ B = X) = $4^n - 3^n$
Subsets of X = 2^n will not repeat in both but here the whole set X has not been taken
So subsets of x which are not repeated $(2^n - 1)$

Hence unordered pair = $\dfrac{(4^n - 3^n) - (2^n - 1)}{2} + (2^n - 1)$

Q.45 **(A)**
Using multinomial theorem
Find the co-efficient of x^{11} in the expansion
$(x + x^2 + x^3 + ...+ x^6)^3 = x^3 (1 - x^6)^3.(1-x)^{-3}$ is
$= {}^{10}C_8 - 3 . {}^{4}C_2 = 45 - 18 = 27$

Q.46 **(D)**

$L_1\ L_3\ L_5$	$L_2\ L_4\ L_6$
$E_1\ E_3\ E_5$	$E_2\ E_4\ E_6$

Number of ways = $3\,!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}\right) . 3\,!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}\right) = 4$

${}^{n-2}C_3 + {}^{n-2}C_2 - {}^{n-2}C_3 = 15$
or ${}^{n-2}C_2 = 15$
$\Rightarrow$ n = 8 $\Rightarrow$ C

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(C,D)**
Number of ways he can fail is either one or two, three or four subject then total of ways.
${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$

Q.2 **(C,B)**
Total no. of visits that a teacher goes is = ${}^{25}C_5$
(selection of 5 different kids each time & teacher goes every time)
Number of visits of a boy = select one particular boy & 4 from rest 24 = ${}^{24}C_4$
So extra visits of a teacher from a boy is = ${}^{25}C_5 - {}^{24}C_4 = {}^{24}C_5$

Q.3 **(A,C,D)**
Total number of required possibilities
${}^5C_3 . {}^8C_7 + {}^5C_4 . {}^8C_6 + {}^5C_5 . {}^8C_5 = {}^5C_3 . {}^8C_7 + {}^5C_4 . {}^8C_6 + {}^8C_5 = {}^{13}C_{10} - {}^5C_3 = 276$

Q.4 **(C,D)**
Here given no. be 1,2,3,.........n
Let common difference = r
Total way of selection = (1, 1 + r, 1+2r),(2, 2 + r, 2 + 2r), ..(n – 2r, n – r, n)
Total numbers are = (n – 2r)
Here $r_{min.} = 1$ and $r_{max.} = (n-1)/2$
Case- I When n is odd

$\therefore\ r_{max} = \dfrac{(n-1)}{2}$ & total no. of selection is

$$= \sum_{r=1}^{(n-1)/2}(n-2r)$$

$$= \frac{n(n-1)}{2} - \frac{2\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)}{2} = \left(\frac{n-1}{2}\right)^2$$

Case - II when n is even = $r_{max} = \frac{n-2}{2}$ so total no. selection is

$$= \sum_{r=1}^{(n-2)/2}(n-2r) = \frac{n(n-2)}{2} - \frac{2\left(\frac{n-2}{2}\right)\frac{n}{2}}{2}$$

$$= \left(\frac{n-2}{2}\right)\left(n-\frac{n}{2}\right) = \frac{n(n-2)}{4}$$

Q.5 **(A,B,C)**

Required number of possible is

$8! - 2.7! = 7!(8-2) = 6.7! \Rightarrow 2.6!.{}^7C_2$

Q.6 **(A,B,C,D)**

(A)Without changing the order of the vowels of MULTIPLE

So we choose the first three place in $^8C_3$ ways and the rest are arranged is

$$\frac{8!}{3!5!} \times \frac{5!}{2!} = \frac{8!}{3!2!} = 3360$$

Hence required no. is 3360 – 1 = 3359

(B) Keeping the position of each vowel fixed M_LT_PL_

Number of ways = $\frac{5!}{2}$ = 60 other ways = 60–1 = 59

(C) without changing the relative order/position of vowels & consonants

so number of ways is = $\frac{5!}{2!} \times 3!$ = 60 × 6 = 360

Hence required number is = 360–1 = 359

(D) Total 8!

Q.7 **(A,B,C,D)**

Value of $^{2n}p_n$ is $\frac{2n!}{n!}$

(i) (n + 1) (n + 2) ----- (2n) and n ! . $^{2n}C_n$

$$\frac{1.2.3.4.5.6.7.8.9.10.11.....(2n-2)(2n-1).2n}{1.2.3........n}$$

$$= \frac{(1.3.5.7....(2n-1)).(2.4.6.8.......2n)}{1.2.3........n}$$

$$= \frac{(1.3.5.7....(2n–1)).2^n(1.2.3.4.......n)}{(1.2.3.4......n)}$$

$= 2^n$ (1.3.5.7. .....(2n – 1)) = (2. 6. 10. 14 ..... (4n – 2))

Q.8 **(A, B, D)**

In (C) number = $^{n+r}C_r$; (D) $^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$]

Q.9 **(B,C,D)**

Given answer is 150 which comes in BCD in A, it is $^5C_3 \cdot 3! = 60$

Q.10 **(B,D)**

Q.11 **(A, B, D)**

(A) 11 $\begin{matrix} / & 4v \\ \backslash & 7c \end{matrix}$

$\Rightarrow {}^7C_2 \cdot 6! = 3 \cdot 7 \cdot 6! = 3 \cdot 7!$

(B) No. of ways = $\frac{15!}{r!(15-r)!} = 15c_v$ $\underbrace{W\,W.....W}_{r}$

$\underbrace{B\,B\,B......B}_{15-r}$

This is maximum if r = 7 or 8

(C) $12 \begin{matrix} \diagup 4A \\ — 5A \\ \diagdown \text{3 all different} \end{matrix}$ $\Rightarrow$ Total no. of combinations

$= 5 \cdot 6 \cdot 2^3 - 1 = 240 - 1 = 239$

(D) 2 alike + 2 other alike + 2 other different = 1

2 alike + 2 other alike + 2 different = $^3C_2 \cdot {}^4C_2$ = 18

2 alike + 4 different = $^3C_1 \cdot {}^5C_1$ = 15

All 6 different = 1

——

= 35 Ans.

Q.12 **(A,B,C)**

(A) 3^5 ; There are 5 pair of numbers having sum equals to 11

(1, 10); (2, 9); (3, 8); (4, 7); (5, 6)

from every pair, one number can be taken in 3 ways

Hence number of subsets $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

e.g. {1, 2, 3, 4, 5}, {1}, {2}, {1, 2}, {ϕ} **etc.**

(B) 3^5 ; xyz = 2310 = $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$

2 can be distributed to x, y, z in 3 ways.

3 can be distributed to x, y, z in 3 ways.

5 can be distributed to x, y, z in 3 ways.

7 can be distributed to x, y, z in 3 ways.

11 can be distributed to x, y, z in 3 ways.

Hence total ways is 3^5

(C) 3^5;

1					

1st place can be filled only in one way i.e. 1

remaining 5 places can be filled in 3^5 ways

Hence total number of ways = $1 \cdot 3^5 = 3^5$

Ans. (D) $3^5 - 1$; Obvious.

Q.13 **(B,C)**

Total required number of teams is

$= {}^{10}C_4 . {}^{6}C_3 . {}^{3}C_3 \cdot \frac{1}{2!}$ = 2100 = $^{10}C_4 . {}^{5}C_2$ = 2100

Q.14 **(B,C,D)**

$$\frac{{}^{200}C_2 . {}^{198}C_2 . {}^{196}C_2 ...... {}^{2}C_2}{100!} = \frac{200!}{2^{100}.100!}$$

$$= \frac{101.102.103.....200}{2^{100}}$$

$$= \left(\frac{100}{2}\right).\left(\frac{102}{2}\right).\left(\frac{103}{2}\right).....\left(\frac{200}{2}\right)$$

And $\frac{1.2.3.4.5.6.7.8.....200}{2^{100}.100!}$

$$= \frac{(1.3.5.7.....199)(2.4.6.8......200)}{2^{100}.100!}$$

$$= \frac{(1.3.5......199).2^{100}.100!}{2^{100}.100!} = 1.3.5\ .199$$

Q.15 **(A,B,D)**

$$\left(\frac{6\,!}{2!\ 2!\ 2!\ 3!}\right) \times 3! = \frac{6!}{2!2!2!} = 90.$$

(A) $\frac{6!}{2!2!2!}$ ⇒ **(A) is correct.**

(B) $^{10}C_2 \cdot 2!$ ⇒ **(B) is correct.**

(C) $5 \begin{cases} 1, 2, 2 \\ 1, 1, 3 \end{cases} = \left(\frac{5!}{1!1!3!}\frac{1}{2!} + \frac{5!}{2!2!1!}\frac{1}{2!}\right)3!$

= 150 ⇒ **(C) is incorrect.**

(Using Division and Distribution)

(D) A and S remains in 2nd and 8th position

Hence, number of ways = $\frac{6!}{2!\,2!\,2!} \cdot 2!$

⇒ **(D) is incorrect.**]

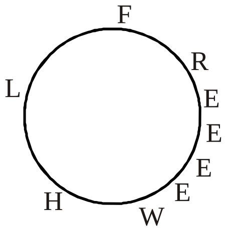
Q.16 **(A,D)**

We have arrange all the letter except ‘CCC’ is $\frac{12!}{5!.3!.2!}$ now there are 13 place where ‘C’ can be placed $^{13}C_3$

Hence required number of ways is = $\frac{12!}{5!\,3!\,2!}\ {}^{13}C_3$

$= 11\ .\ \frac{13!}{6!}$

Q.17 **(B,C,D)**



(A) False as is should be $^{9}P_5 - 1$

(B) x . 4! = 8!

∴ $x = \frac{8!}{4!} = {}^{8}P_4$

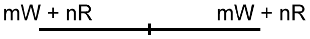
(C) Vowels E E E E select 4 places in $^{9}C_4$ ways arrange consonants alphabatically only us one ways.

∴ $^{9}C_4 = 126 = \frac{1}{2} \cdot 256 = \frac{1}{2} \cdot {}^{10}C_5$

(D) True

∴ correct answer are (B), (C) and (D)

Q.18 **(A,B)**



S_2 : Arrangements will be one side

∴ $^{m+n}C_m$

Q.19 **(A,B,C,D)**

A			

4! = 24 ways

G				

4!/2! = 12 ways

I				

$\frac{4!}{2!}$ = 12 ways

N	A	A	G	I

and

N	A	A	I	G

here 50th

NAAIG

Q.20 **(A,C,D)**

B U L B U L; number of ways = $\frac{6!}{2!\cdot2!\cdot2!}$

(A) 2 Apples can be distributed in 3 people in $^{4}C_2$ way O O Ø Ø

and 4 Mangoes in $^{6}C_2$ ways

O O O O Ø Ø

∴ Total ways = $^{6}C_2 \cdot {}^{4}C_2 = \frac{6!}{2!\cdot4!} \cdot \frac{4!}{2!\cdot2!}$

$= \frac{6!}{2!\cdot2!\cdot2!}$ ⇒ **(A)** is correct

(B) 6 books in 3 bundles, two in each bundle

$= \dfrac{6!}{2!\cdot 2!\cdot 2!\cdot 3!}$ ⇒ **(B)** is incorrect

(C) T_{r+1} in $(x + y + z)^6$ is ${}^6C_r(x + y)^{6-r}\cdot z^r$

put $r = 2$

$T_3 = {}^6C_2(x + y)^4 \cdot z^2$

$= {}^6C_2 \cdot z^2\ ({}^4C_p \cdot x^{4-p} \cdot y^p)$

put $p = 2$

$= {}^6C_2 \cdot z^2 \cdot {}^4C_2 \cdot x^2y^2)$

$= {}^6C_2 \cdot {}^4C_2\ x^2y^2z^2$

$= \dfrac{6!}{2!\cdot 2!\cdot 2!}$

⇒ **(C)** is correct

(D) 6 prizes in 3 childrents, two to each $= \dfrac{6!\cdot 3!}{2!\cdot 2!\cdot 2!\cdot 3!}$

$= \dfrac{6!}{2!\cdot 2!\cdot 2!}$ ⇒ **(D)** is correct]

Q.21 **(B,C,D)**

| T | M | A | T |

Number of ways $= {}^5C_2 \times \dfrac{4!}{2!} = 10 \times 12 = 120.$

(A) RAITHATHA

A = 3, H = 2, T = 2, R = 1, I = 1

$\boxed{A\ A\ A}\quad \boxed{H\ H}\quad \boxed{T\ T}\quad R\quad I$

∴ Number of ways = 5! = 120

(B)
2, 3, 3
3, 2, 2
2, 3, non-zero digit
2, 3, 0

$= \dfrac{3!}{2!} + \dfrac{3!}{2!} +$

$({}^7C_1 \times 3!) + (2 \times 2) = 3 + 3 + 42 + 4 = 52.$

(C) $M_1, M_2, \ldots\ldots\ldots, M_7 \mid N_1, N_2, N_3$

Total ways $= 3\ ({}^7C_1) + 3({}^7C_2) + 1\ ({}^7C_3) = 21 + 63 + 35 = 119.$

Alternatively:

Number of ways $= {}^{10}C_3 - 1 = 120 - 1 = 119$ **Ans.**

(D) Number of ways $= {}^4C_2 \times {}^6C_2 = 6 \times 15 = 90.$ **Ans.**

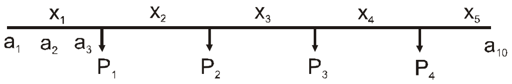
Q.22 **(B,D)**

$x_1 + x_2 + x_3 + x_4 \le n \Rightarrow x_1 + x_2 + x_3 + x_4 + y = n$

(where y is known as pseudo variable)

Total no. of required solution is $= {}^{n+5-1}C_n = {}^{n+4}C_n$ or ${}^{n+4}C_4$

Q.23 **(B,C,D)**



$x_1 + x_2 + x_3 + x_4 + x_5 = 6 \Rightarrow x_1 + y_1 + y_2 + y_3 + x_5 = 3$ but $x_1, x_5 \ge 0$

$x_2, x_3, x_4 \ge 1 \Rightarrow y_1, y_2, y_3 \ge 0$

${}^{3+5-1}C_3 \,.\, 4\,! = {}^7C_3 \,.\, 4\,! = {}^7p_3 \,.\, 4 = 840$

Q.24 **(A,D)**

Consider $(x + y + z)^9 = {}^9C_r\, x^{9-r}\, (y + z)^r = {}^9C_r\, x^{9-r} \cdot {}^rC_p\, y^{r-p}\, z^p\ [x + (y + z)]^9$

put $r = 7\ ;\ p = 4$

or ${}^9C_2 \cdot {}^7C_3 \cdot {}^4C_4 = \dfrac{9!}{2!\,3!\,4!}$

Q.25 **(A,B)**

$\dfrac{50!}{14!36!}$ exp of 19 in 50 ! $= \left[\dfrac{50}{19}\right] + \left[\dfrac{50}{19^2}\right] = 2$

Exp. of 19 in 36 ! $= \left[\dfrac{36}{19}\right] + \left[\dfrac{36}{19^2}\right] = 1$

⇒ ${}^{50}C_{36}$ is divisible by 19 but not by 19^2

Exp. of 5 in 50 ! $= \left[\dfrac{50}{5}\right] + \left[\dfrac{50}{25}\right] = 12$; Exp. of 5 in 14 ! $= \left[\dfrac{14}{5}\right] = 2$

Exp. of 5 in 36 ! $= \left[\dfrac{36}{5}\right] + \left[\dfrac{36}{25}\right] = 8$

Ans. A & B

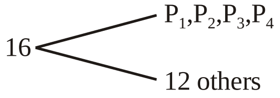
Comprehension # 01 (Q. 26 to Q.28)

26. (A)
27. (C)
28. (D)

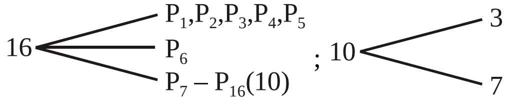
$$\text{Number of ways} = \frac{(16)!}{(4!)^4\,4!} = \frac{2^8 \cdot 8! \prod_{r=1}^{8}(2r-1)}{4!\cdot 4!\cdot 4!\cdot 4!\cdot 4!}$$

$$= \frac{2^8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \lambda}{(24)\cdot(24)\cdot(24)\cdot(24)} = \frac{2^9 \cdot 35 \cdot \lambda}{(24)\cdot(24)\cdot(24)}$$

$$= \frac{35}{27}\prod_{r=1}^{8}(2r-1) \text{ Ans.}$$



12 others can be divided into 4 equal groups in each of 3 person $\dfrac{(12)!}{(3!)^4} = \dfrac{(12)\cdot(11)!}{6\cdot 6\cdot 6\cdot 6} = \dfrac{(11)!}{108}$



$$= \frac{10!}{3!\,7!}$$

$$\underbrace{\underbrace{0\ 0\ 0}_{+P_6}}_{4 \text{ players}} \quad \underbrace{\boxed{\underbrace{x\,x\,x\,x\,x\,x\,x\,x}_{P_1 \text{ to } P_5}}}_{12 \text{ players}}$$

Now, $12 \begin{matrix} \nearrow 4 \\ \rightarrow 4 \\ \searrow 4 \end{matrix} = \dfrac{12!}{(4!)^3 \cdot 3!}$

$$\text{Total ways} = \frac{(10)!}{3!\,7!} \cdot \frac{(12)!}{(4!)^3 \cdot 3!} = \left(\frac{12!}{(4!)^3}\right) \cdot 20$$

$\Rightarrow$ k = 20 **Ans.**

Comprehension # 02 (Q. 29 to Q.31)

29. (C)

30. (A)

31. (B)

Vowels" I = 3; A = 1; O = 2

Consonants: S = 1; L = 1 : C = 1 ; N = 1 ; T = 2

Vowels appear in alphabetical order

$\therefore\ {}^{12}C_6 \cdot 1 \cdot \dfrac{6!}{2!} \ \Rightarrow \dfrac{12!}{6!\,2!} = \dfrac{12 \times 11 \times 10!}{12 \times 5!} = 66 \cdot 7!$

$\Rightarrow$ K = 66 **Ans.**

2 consonants can be selected in

$^2C_2 + {}^5C_2 = 1 + 10 = 11$

number of arrangement of 2 consonants

$= 1 + 10 \times 2! = 21$

2 vowels can be selected in

$^2C_1 + {}^3C_2 = 5$

and arrangement is 2 + 3 × 2! = 8

$\therefore$ Number of 4 letter words

$^4C_2 \times 21 \times 8 = 6 \times 21 \times 8 = 126 \times 8 = 1008$ **Ans.**

Number of words when I's separated

$$\frac{9!}{2!\,2!} \cdot {}^{10}C_3 \cdot 1$$

and number of words when I's separated and O's together

$$\frac{8!}{2!} \cdot {}^9C_3 \cdot 1$$

$$\therefore \text{Total} = \frac{9!}{2!\,2!} \cdot {}^{10}C_3 - \frac{8!}{2!} \cdot {}^9C_3 = 228 \times 8!$$

$\Rightarrow$ N = 228 **Ans.**]

Comprehension # 03 (Q. 32 and Q.33)

Q.32 (B)

3 official out of 8 can be selected by $^8C_3 = 56$ ways

2 non-official out of 4 can be selected in $^4C_2 = 6$ ways

$\therefore$ required number of committees are 56 × 6 = 336.

Q.33 **(A)**

Two non-officials and 3 officials i.e.

$^4C_2 \times {}^8C_3 = 6 \times 56 = 336.$

Three non-official and 2 officials

$^4C_3 \times {}^8C_2 = 4 \times 28 = 112.$

Four non-officials and 1 official

$^4C_4 \times {}^8C_1 = 1 \times 8 = 8$

Total 336 + 112 + 8 = 456.

34. **(D)**

Required no. of ways

$= {}^{12-1}C_5 = {}^{11}C_5 = 462$

Comprehenssion # 04 (Q. No. 35 and 36)

Q.35 (B)

Q.36 **(A)**

In the word RESONANCE there are 9 letters.

Consonants (5), 1R, 1S, 1C and 2N

Vowels (4), 2E, 1O, 1A

total even places 4 ;

No. of ways arranging vowels in even places is $\dfrac{4!}{2!}$ = 12

No. of ways arranging consonants in remaining odd places is $\dfrac{5!}{2!}$ = 60

required number of arrangement = 12 × 60 = 720 = n

Required number of arrangements are $\dfrac{9!}{2!\,2!\,4!}$ = 3780

Comprehension # 05 (Q. No. 37 and 38)

Q.37 (D)

Q.38 **(D)**

Let n lines divides the pizza into S_n pieces. Let us add new (n + 1)th line, L which cuts the previous n lines by assumption. Now line L will cut the original pieces into 2 pieces further & we are passing trough (n + 1) such pieces, hence

$S_{n+1} = S_n + (n + 1)$

$S_n = \dfrac{n(n+1)}{2} + 1$

when $S_n \geq 60$ where $n \in N$

$n(n + 1) + 2 \geq 60$

$n^2 + n - 11 \geq 0$

n = 10, is not satisfy

n = 11 is satisfying

$\Rightarrow$ n = 11

MATCH THE COLUMN:

Q.39 (A) R; (B) S; (C) Q; (D) P

(A) From n elements select m in ${}^{n}C_{m}$ ways and can be arranged only in one way.

(B) 1st working can be given to $M_1\ M_2\ M_3 \ldots\ldots\ldots M_m$ ® 'm' men

any one of m must $k_1\ k_2\ k_3 \ldots\ldots k_n \rightarrow$ 'n' monkey

|||ly 2nd, 3rd , ..., nth monkey in *m* way

total ways = m^n

(C) Number of gaps = m

select *n* gaps in ${}^{m}C_{n}$ ways for *n* red balls $\Rightarrow$ total ways = ${}^{m}C_{n}$

(D) 1st toy in *n* ways

2nd toy in *n* ways

and so on. Total ways = n^m

Q.40 (A) Q; (B) Q; (C) S ; (D) P; (E) Q

(A) 3 right and 2 wrong + 3 wrong and 2 right + all five right

$${}^{5}C_2 \cdot 2 \quad + \quad {}^{5}C_3 \cdot 1 \quad + \quad 1 \quad = 31$$

(B) $N = 2^2 \cdot 3^2 \cdot 5 \cdot 11$

Total number of divisors = $3 \cdot 3 \cdot 2 \cdot 2 = 36$

prime divisors of N are 2, 3, 5, 11 = 4 prime divisors

number of divisors that are composite = 36 – 5 = 31]

(C) Exactly one 2's $= {}^{7}C_1$ | × | × | × | × | × | × | (×'s indicate 3 and bar indicates gap)

Exactly two 2's $= {}^{6}C_2$ | × | × | × | × | × |

Exactly three 2's $= {}^{5}C_3$ | × | × | × | × |

Exactly four 2's $= {}^{4}C_4 = 1$ | × | × | × |

Total = 7 + 15 + 10 + 1 = 33 Ans.

(D) Peaches 5 p_1, p_2, p_3, p_4, p_5

Apples 3 a_1, a_2, a_3

Hence number of ways $= {}^{3}C_1 \times {}^{5}C_3 = 30$ **Ans.**

(E) Case I ${}^{6}C_3$; Case II AAA BBB $\Rightarrow$ (all three alike $\Rightarrow$ 2 ways

or 2 alike and 1 different $\Rightarrow$ 2 ways = 4 ; Case III AA BB CC all three different

1 ; 2 alike + 1 different = 3 . 2 = 6 $\Rightarrow$ Total ways = 20 + 4 + 7 = 31)

Q.41 (A) Q, (B) S, (C) T, (D) P

(A) C_6 can be seated any where in the row. For C_5 there are two options one just before C_6 and one just after C_6. similarly for C_4 also there are 2 options $\Rightarrow$ Total $2^5 = 32$ **Ans.**

(B) $\underbrace{{}^{6}C_3}_{\textit{select 3 places}} \cdot 1 \cdot 3! - {}^{6}C_4 \cdot 2! = 120 - 30 = 90$ **Ans.**

Alternatively : ${}^{6}C_4 \cdot 3 \cdot 2! = 15 \cdot 3 \cdot 2 = 90$ **Ans.**

(C) Give one to each of the 6 children. Now 4 remaining can be given as follows:

(i) 1 marble to each of 4 children $\Rightarrow$ ${}^{6}C_4 = 15$ ways

(ii) 2 marbles to each of two children $\Rightarrow$ ${}^{6}C_2 = 15$

(iii) 3 marbles to one and 1 to other of five $= {}^{6}C_1 \cdot {}^{5}C_1 = 30$

(iv) 1 marbles to two and 2 marbles to 1 child ${}^{6}C_2 \cdot {}^{4}C_1 = 60$

So, total ways = 120 **Ans.**

(D) Select 3 out of six in ${}^{6}C_3$ ways for one column and 3 rejected children on the other column can be arranged only in 1 way

Hence number of ways ${}^{6}C_3 \cdot 1 = 20$ ways **Ans.**]

Q.42 (A) $\rightarrow$ (R), (B) $\rightarrow$ (P), (C) $\rightarrow$ (Q), (D) $\rightarrow$ (S)

(A) Required number of ways = (2 + 1) (3 + 1) (4 + 1) – 1 = 59.

(B) ${}^{10}C_3 = 120$

(C) Requied number of ways

= Coefficient of x^{10} in $(1 + x + x^2 + \ldots\ldots)^4$

= Coefficient of x^{10} in $(1 - x)^{-4} = {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

(D) The word ‘MATHEMATICS’ consists of 11 letters of which 7 are consonants namely M,M, T, T,H,C,S and 4 vowels and a group of consonants can be

arranged in $\dfrac{{}^{5}P_5}{2!} = \dfrac{5!}{2}$ ways. ($\therefore$ A is repeated twice)

In any such arrangement, seven consonants can be reshuffled among themselves in

$\dfrac{{}^{7}P_7}{2!\ 2!} = \dfrac{7!}{2!\ 2!}$ ways.

($\because$ Either of M and T is repeated twice)

Hence, the required number of ways $= \dfrac{5!}{2!} \times \dfrac{7!}{2!\,2!}$

$= \dfrac{120 \times 5040}{8} = 75600.$

Q.43 (A) - (Q) ; (B) - (P) ; (C) - (S) ; (D) - (R)

(A) ${}^{8}C_4 + {}^{8}C_3 \times {}^{5}C_1 + {}^{8}C_2 \times {}^{5}C_2$

(B) The number of ways of selecting 3 points out of 12 points is ${}^{12}C_3$.

Three points out of 7 collinear points can be selected in ${}^{7}C_3$ ways.

Hence, the number of triangles formed is ${}^{12}C_3 - {}^{7}C_3 = 185$.

(C) ${}^{m}C_2 \times {}^{n}C_2$

(D) Two circles intersect in 2 points.

$\therefore$ Maximum number of points of intersection of two circles = 2 × number of selections of two circles from 8 circles.

$= 2 \times {}^{8}C_2 = 2 \times 28 = 56$

$\therefore$ Maximum number of points of intersection of two straight line = 1 × number of selections of two
straight line from 4 straight line = ${}^{4}C_2 = 6$

$\therefore$ Maximum number of points of intersection of one straight line and one circle = 2 × number of selections of one straight line from 4 straight line and number of selections of one circles from 8 circles

$= {}^{4}C_1.{}^{8}C_1.2 = 64$

NUMERICAL VALUE BASED

Q.1 (15)

Number divisible by 3 if sum of digits divisible

case-I If 1 + 2 + 3 + 4 + 8 = 18

Number of ways = 120

case-II If 1 + 2 + 3 + 7 + 8 = 21

Number of ways = 120

case-III If 2 + 3 + 4 + 7 + 8 = 24

Number of ways = 120

case-IV If 1 + 2 + 0 + 4 + 8 = 15

Number of ways = 96

case-V If 1 + 2 + 0 + 7 + 8 = 18

Number of ways = 96

case-VI If 2 + 0 + 4 + 7 + 8 = 21

Number of ways = 96

case-VII If 0 + 1 + 3 + 4 + 7 = 15

Number of ways = 96

____

total number

744

Q.2 (0)

Let the number starts with 635 then two cases arise

Case-1 If 9 occurs at units place then the number of numbers = 9 × 9 × 9 = 729

Case-2 If 9 does not come at units place then number of ways for 9 to occur at either of the rest three places = ${}^{3}C_1 = 3$

$\therefore$ the number of numbers = 3 × 9 × 9 × 4 = 972

$\therefore$ total numbers starting with 635 = 729 + 972 = 1701

Similarly total number starting with 674 = 1701

$\therefore$ Maximum number of trials = 1701 × 2 = 3402

Q.3 (10)

Ten digits can be partitioned into four parts as

1 + 1 + 3 + 5 ; 1 + 1 + 1 + 7 ; 1 + 3 + 3 + 3

(each partitioning has odd number of digits)

The number of ways in which these can be placed in the four spaces =

$$\frac{4!}{2!} + \frac{4!}{3!} + \frac{4!}{3!} = 20 \text{ ways}$$

also numbers of arrangements of vowels = 5 !

Number of arrangements of digits = 10 !

total ways = 20 (10 !) (5 !)

Q.4 (48)

8 - non identical

number of ways =

B_1	B_2	B_3
1	3	4
2	3	3
2	2	4

Here required number of ways = 3!

$$\left\{{}^{8}C_1 \,.\, {}^{7}C_3 . {}^{4}C_4 + \frac{{}^{8}C_2.{}^{6}C_3.{}^{3}C_3}{2!} + \frac{{}^{8}C_2.{}^{6}C_2.{}^{4}C_4}{2!}\right\}$$

$$= 6\left[\frac{8!}{3!.4!} + \frac{8!}{2!.6!} \times \frac{6!}{3!.2!.2!} + \frac{8!}{2!.6!} \times \frac{6!}{2!.4!.2!}\right]$$

= 6[280 + 280 + 210] = 6 × 770 = 4620

Q.5 (2)

Find the coefficient of x^{14} in the expansion of

$(x^0 + x^2 + x^4)^5 = (1 + x^2 + x^4)^5 = \left(\frac{1-x^6}{1-x^2}\right)^5 = (1 - x^6)^5$

$(1 - x^2)^{-5}$

$= (1 - 5x^6 + 10x^{12}............)$

$\left(1 + {}^{5}C_1x^2 + {}^{6}C_2x^4 + {}^{7}C_3x^6 + ...\right) = {}^{11}C_7 - 5 \,.\, {}^{8}C_4 + 10.5$

$= \frac{11\times10\times9\times8}{1\times2\times3\times4} - 5.\frac{8\times7\times6\times5}{1\times2\times3\times4} + 50 = 330 - 350 + 50 = 30$

Q.6 (42)

SERIES

S - 2, E - 2 , R, I

case-I when all letter distinct is

${}^{4}C_3 \times 3! = 4 \times 6 = 24$

case-II when 2 letters are same the

$${}^{2}C_1 \,.\, {}^{3}C_1 \times \frac{3!}{2!} = 2 \,.\, 3 \,.\, 3 = 18$$

total number is 24 + 18 = 42

Q.7 (4)

Case -I If all are different then no. of ways is = ${}^{6}C_3 = 20$

Case-II If three each of two colours, then combination is

3 0 $\to$ 2!
2 1 $\to$ 2!
= 2! + 2! = 4 ways

Case-III If two each of three colours, then combination is

2 1 0 $\to$ 3!
1 1 1 $\to$ 1! = 3! + 1! = 7ways

Hence required no.is = 20 + 7 + 4 = 31

Q.8 (12)

$$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{2} = \frac{5!}{2!3!} = 10$$

$$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{3} = \frac{5!}{3!} = 20$$

$$\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{4} = \frac{5!}{2!2!} = 30$$

Hence sum of unit places is

$2 \times 10 + 3 \times 20 + 4 \times 30 = 200$

Hence required sum is

$= 200 \times (10^5 + 10^4 + 10^3 + 10^2 + 10^1 + 10^0)$

$= 200 \times (111111) = 22222200$

Q.9 (20)

First we select one married couple out of 6 married couple i.e. ${}^6C_1$ ways

total number of required case ${}^6C_1 \times {}^5C_1 \times {}^4C_1 \times 2 = 6 \times 5 \times 4 \times 2 = 240$

N = 240

Q.10 (8)

Using multinomial theorem total number of required selection is

${}^{8+3}C_8 = {}^{11}C_8 = {}^{11}C_3 = 165$

KVPY

PREVIOUS YEAR'S

Q.1 (B)

$$\frac{6!}{(2!)^3} = 90$$

Q.2 (A)

Total Integers = 999 – 99 = 900

Total Integers in which all distinct digits

$\boxed{9}\boxed{9}\boxed{8} = 648$

So 900 – 648 = 252

Q.3 (A)

EDUCATION
Vowels EUAIO
Consonant DCTN
$= 1 \times {}^6C_4 \times 1 = 15$

Q.4 (B)

Let Red Balls = x
White Balls = y
Blue Balls = z
Green Balls = w

$$\frac{{}^xC_4}{{}^{x+y+z+w}C_4} = \frac{{}^xC_3 \cdot {}^yC_1}{{}^{x+y+z+w}C_4} = \frac{{}^xC_2\,{}^yC_1\,{}^zC_1}{{}^{x+y+z+w}C_4} = \frac{{}^xC_1 \times {}^yC_1 \times {}^zC_1 \times {}^wC_1}{{}^{x+y+z+w}C_4}$$

${}^xC_4 = {}^xC_3\,{}^yC_1$ $\quad$ $x - 3 = 4y$

$x = 4y + 3$

${}^xC_3\,{}^yC_1 = {}^xC_2\,{}^yC_1\,{}^zC_1$ $\quad$ $x - 2 = 3z$

$x = 3z + 2$

${}^xC_2\,{}^yC_1\,{}^zC_1 = {}^xC_1\,{}^yC_1\,{}^zC_1\,{}^wC_1$ $\quad$ $x - 1 = 2w$

$x = 2w + 1$

Cleary for y = 1 not possible

at y = 2 x = 11
z = 3 x = 11
w = 5 x = 11

So, minimum number of Ball = 11 + 2 + 3 + 5 = 21

Q.5 (C)

When n = 10

Let A_r be no. of ways of selecting r numbers.

No. of selection of A is

$= n(A_0) + n(A_1) + n(A_2) + n(A_3) + n(A_4)$

$= 1 + 10 + (7 + 6 + 5 + \ldots + 1) + (4 + 3 + 2 + 1) + (3 + 2 + 1) + (2 + 1) + 1$

$$= 11 + \frac{7.8}{2} + 10 + 6 + 3 + 1 + 1 = 60$$

N(p) = n(no. of ways 1 is selected) = 1 + 7 + 4 + 3 + 2 + 1 + 1 = 19

N(q) = n(no. of ways 2 is selected) = 1 + 6 + 3 + 2 + 1 = 13

So $p = \dfrac{19}{60}$ $\quad$ $q = \dfrac{13}{60}$

$$p - q = \frac{1}{10}$$

Q.6 (B)

We need 3-digit number which is divisible by 4 & 5 both.

i.e. their last two digits are

00, 20, 40, 60, & 80

Now, ending with 00 are '9'

{100, 200, ..., 900}

but 220 numbers can be permuted according to the condition as {220, 202}

So, there are '8' other favorable cases.

If the number have no digit repeated like 320.

320 can be permuted in 4 ways.

{302, 230, 320, 203}

So, such numbers are 8 × 4 × 4 = 128

Total favorable = 9 + 8 + 128 = 145

So, required prob $\dfrac{145}{900} = \dfrac{26}{180}$

Q.7 (C)

Number of scalene triangles

$$\begin{aligned}&= {}^{13}C_3 - 3\left\{\begin{matrix}10, & 11, & 22\\ 10, & 12, & 22\\ 10, & 11, & 21\end{matrix}\right\}\\ &= 283\end{aligned}$$

Number of isosceles triangles

$$\begin{aligned}&= \left({}^{13}C_2 \times 2\right) - 4\left\{\begin{matrix}10, & 10, & 22\\ 11, & 11, & 22\\ 10, & 10, & 21\\ 10, & 10, & 20\end{matrix}\right.\\ &= 152\end{aligned}$$

Number of equilateral triangels
$= {}^{13}C_1 = 13$
So total number of triangels = 448

Q.8 (B)

ways = $3^4 - {}^3C_1 \cdot 2^4 + {}^3C_2 \cdot 1^4 = 81 - 48 + 3 = 36$.

Q.9 **(A)**

Let the number be N

If we write c after the last digit now number is 10N + c

Now $c|10N + c\ \ \forall\ c = 1, 2, ..., 9$

$\Rightarrow\ c|10N\ \ \forall\ \ c = 1, 2, ..., 9$

$\Rightarrow\ c|10N$ for c = 4, 7, 9

Hence N is LCM of (4, 7, 9) = 252

so sum of digit = 9

Q.10 **(D)**

Since there exists a prime in every set of the form {10a + b, a = 1, 2, ….,9, b = 0, 1, 2,….,9}

Hence every two digit number is almost prime.

Q.11 **(B)**

$$\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 8\ 9}_{9-\text{digit}}\underbrace{10\ \ 11\ \ 12\ \ \ldots\ldots 99}_{9-\text{digit}}\ \ 100\ \ 101$$

Wee need 2021st digit

Till two digit number we have 189 digit we need 2021 – 189 = 1832 digit

triple digit $\dfrac{1832}{3} = 610 \times 3 + 2$

we take 610 three digit number

100, 101, ..., 709

$$\underbrace{1\ 2\ 3\ \ldots\ldots 9}_{9\text{ digit}}\ \ \underbrace{10\ 11\ \ 12\ \ 13\ \ \ldots\ldots 99}_{180\text{ digit}}\ \ \underbrace{100\ \ 101\ \ \ldots. 709}_{1830\text{ digit}}\ \ \underset{\substack{\downarrow\\ 2020^{st}\text{digit}}}{71}\qquad 2021^{st}\text{digit}$$

Total = 9 + 180 + 1830 = 2019

Ans = 1

Q.12 **(B)**

(When a_1 does not occupy its position) its position but a_2

(When a_1 does not occupy its position but a_2 occupy its second positon)

4	×	4!	–	3	×	3!	=	78
a_1 can occupy any position except Ist		Remaing 4-letter can be arranged in 4-position		a_1 can occupy 3-position except Ist and IInd		Remaining threeperson arrange in 3-position		

Q.13 **(B)**

Put all block cover books together

$A_1A_{12}A_3\ldots..A_m \rightarrow \alpha$

Total number of books = n + 1

These books be arranged in (n + 1)! ways

and m books be arranged m m! ways

No. of way = m! (n + 1)!

Q.14 **(B)**

abcde × 9 = edcba

surely a = 1

$\Rightarrow$ 1bcde × 9 = edcb1

9e last digit is 1 $\Rightarrow$ e = 9

$\Rightarrow$ 1bcd9 × 9 = 9dcb1

9 multiply by b $\Rightarrow$ b has to {0, 1} otherwise RHS is a six digit number

C-1 Take b = 0

10cd9 × 9 = 9dc01

9d + 8 = P0 $\rightarrow$ (last digit has to be zero)

$\Rightarrow$ d = 8

10c89 × 9 = 98c01

Now 98c01 is divisible by 9 $\Rightarrow$ sum of digit divisible by 9 $\Rightarrow$ c = 0, 9

take c = 0, 10089 × 9 = 90801 (rejected)

take c = 9, 1089 × 9 = 9801

a = 1, b = 0, c = 9, d = 8, e = 9

Sum = 27

C-2 take b = 1

11cd9 × 9 = 9dc11

9d + 8 = p1 $\Rightarrow$ d = 7

9 × 11c79 = 97c11

This cannot be true for $c \in \{0, 1, 2, ..., 9\}$

Alternate Solution

$9(a \times 10^4 + b \times 10^3 + c \times 10^2 + b \times 10 + e)$

$= e \times 10^4 + d \times 10^3 + c \times 10^2 + b \times 10 + a$

89999a + 8990b + 800c – 910d – 9991e = 0

for max. value of ‘a’

put b = c = 0 and d = e = 9

$a = \dfrac{98109}{89999} \Rightarrow$ a will be 1

$\therefore$ 89999 + 8990b + 800c – 910d – 9991e = 0

for max, value of b

put c = 8 & d = e = 9

$\therefore b = \dfrac{8110}{8990} \Rightarrow$ b will be 0

$\therefore$ 8999 + 800 c – 910d – 9991e = 0

for max. value of c

put d = e = 9 $\Rightarrow$ c > 10 (not possible)

put d = e = 8 (not possible)

put d = 9, e = 9 (not possible)

put d = 8, e = 9

$\Rightarrow c = \frac{7200}{800} \Rightarrow c = 9$

$\therefore$ number is 10989

JEE MAIN
PREVIOUS YEAR'S

Q.1 (2)

$\boxed{}\boxed{}\boxed{}$ divisible by → 3 divisible by 5

$\boxed{}\boxed{}\boxed{5}$ = 12

4 × 3

12 → 3, 4, 5 → 3! = 6

15 → 2, 3, 4 → 3! = 6

24 → 1, 3, 5 → 3! = 6

42 → 1, 2, 3 → 3! = 6

$\overline{24}$

Required No. = 24 + 12 – 4 = 32

Q.2 (31650)

A	B	C
1	8	1
2	7	1
⋮	⋮	⋮
6	1	3

Ways to distribute in groups = $^{10}C_1(^9C_1 + ... ^9C_8) + {}^{10}C_2(^8C_1 + ...+ {}^8C_7) + {}^{10}C_3(^7C_1 +.....+ {}^7C_6)$

= 10(510) + 45(254) + 120(126)

= 31650

Q.3 (1)

Case-1: 1, 1, 1, 1, 1, 2, 3

ways = $\frac{7!}{5!} = 42$

Case-2: 1, 1, 1, 1, 2, 2, 2

ways = $\frac{7!}{5!.3!} = 35$

total ways = 42 + 35 = 77

Q.4 (1000)

Number must be an odd multiple of 3 and not a multiple of 9

4-digit odd multiples of 3 are

1005, 1011, ...9999 → 1499

4-digit odd multiples of 9 are

1017, 1035, ..., 9999 → 499

$\therefore$ Required numbers →1000

Q.5 (3)

Solving given two equation we get y=3, z=2

$\Rightarrow N = 2^x\, 3^3\, 5^2$

number of odd divisor = (2+1) (3+1)=12

Q.6 (1)

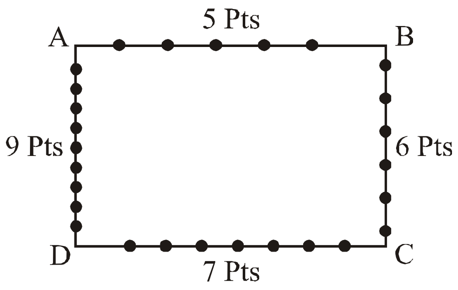
(2I, 4F) + (3I, 6F) + (4I, 8F)

$= {}^6C_2\ {}^8C_4 + {}^6C_3\ {}^8C_6 + {}^6C_4\ {}^8C_8$

= 15 × 70 + 20 × 28 + 15 × 1

= 1050 + 560 + 15 = 1625

Q.7 **(4)**



α = Number of triangles

$\alpha = 5{\cdot}6{\cdot}7 + 5{\cdot}7{\cdot}9 + 5{\cdot}6{\cdot}9 + 6{\cdot}7{\cdot}9$

= 210 + 315 + 270 + 378

= 1173

β = Number of Quadrilateral

$\beta = 5{\cdot}6{\cdot}7{\cdot}9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

Q.8 **(3)**

Total matches between boys of both team

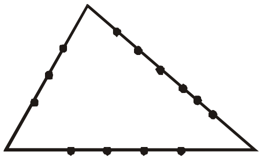
$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both team $= {}^nC_1\ {}^6C_1 = 6n$

Now, 28 + 6n = 52

$\Rightarrow n = 4$

Q.9 **(3)**



Total Number of triangles formed

$= {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3$

= 333 **Option (3)**

Q.10 Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

So, sum = $(3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

= (1111) × 24

= 26664

Q.11 **(300)**

3_ _ = 10 × 10 = 100

_3 _ = 10 × 10 = 100

_ _ 3 = 10 × 10 = $\frac{100}{300}$

Q.12 **(96)**
Q.13 **(1251)**
Q.14 (238)
Q.15 [777]
Q.16 (77)
Q.17 [576]
Q.18 **(7744)**
Q.19 (136)
Q.20 (52)
Q.21 [25]

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(B)**

	B_1	B_2	B_3
Case-1:	1	1	3
Case-2:	2	2	1

Ways of distribution $= \frac{5!}{1!\,1!\,3!\,2!}$. 3! +

$\frac{5!}{2!\,2!\,1!\,2!}$ .3! = 150

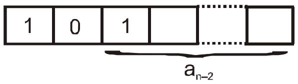
Paragraph for Question Nos. 2 & 3

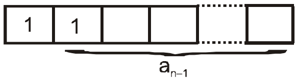
Let a_n denote the number of all n-digit positive integers formed by the digits 0,1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

Q.2 **(A)**

$\underline{1}$----------------- $\underline{1}$ # a_{n-1}
----------------- $\underline{1}$ $\underline{0}$ # a_{n-2}
So $a_n = a_{n-1} + a_{n-2}$
So A choice is correct
consider B choice $c_{17} \neq c_{16} + c_{15}$
$c_{15} \neq c_{14} + c_{13}$ is not true
consider C choice $b_{17} \neq b_{16} + c_{16}$
$a_{16} \neq a_{15} + a_{14}$ is not true
consider D choice $a_{17} = c_{17} + b_{16}$
$a_{17} = a_{15} + a_{15}$ which is not true

Aliter





using the Recursion formula
$a_n = a_{n-1} + a_{n-2}$
Similarly $b_n = b_{n-1} + b_{n-2}$ and $c_n = c_{n-1} + c_{n-2}$ $\forall$ $n \geq 3$
and $a_n = b_n + c_n$ $\forall$ $n \geq 1$
so $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 5$, $a_5 = 8$..........
$b_1 = 1$, $b_2 = 1$, $b_3 = 2$, $b_4 = 3$, $b_5 = 5$, $b_6 = 8$ ....
$c_1 = 0$, $c_2 = 1$, $c_3 = 1$, $c_4 = 2$, $c_5 = 3$, $c_6 = 5$ ....
using this $b_{n-1} = c_n$ $\forall$ $n \geq 2$

Q.3 **(B)**

$b_6 = a_5$
$a_5 = \underline{1}$ - - - $\underline{1}$ $\underline{1}$ - - - $\underline{0}$
${}^3C_0 + {}^3C_1 + 1 + {}^2C_1 + 1$
1 + 3 + 1 + 2 + 1
4 + 4 = 8

Q.4 **(7)**

$n_2 = n_1 + t_1 + 1$
$n_3 = n_2 + t_2 + 1$
$n_4 = n_3 + t_3 + 1$
$n_5 = n_4 + t_4 + 1$
The given equation becomes
$5n_1 + 4t_1 + 3t_2 + 2t_3 + t_4 = 10$

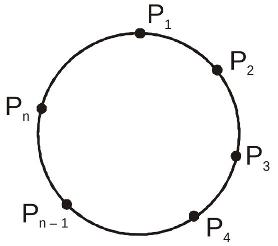
where $n_1 \geq 1$; $t_1 \geq 0$
$n_1 = t_0 + 1 \Rightarrow 5t_0 + 4t_1 + 3t_2 + 2t_3 + t_4 = 5$
$t_0 = 1$ will yield only 1 solution.
so $t_0 = 0$,
$4t_1 + 3t_2 + 2t_3 + t_4 = 5$.
$t_1 = 0 = t_2$. there will be 3 solution
$t_1 = 0$, $t_2 = 1$ will yield 2 solution.
$t_1 = 1$, t_2 must be zero 1 solution.
Hence in total there will be 7 solution.

Alternative :

n_1	n_2	n_3	n_4	n_5
1	2	3	4	10
1	2	3	5	9
1	2	3	6	8
1	2	4	5	7
1	2	4	6	8
1	3	4	6	7
2	3	4	5	6

Q.5 **(5)**

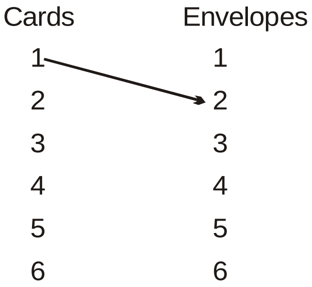
Number of adjacent lines = n
Number of line segment joining non-adjacent points is ${}^nC_2 - n$.



Now, n = $({}^nC_2 - n) \Rightarrow 2n = \frac{n(n-1)}{2} \Rightarrow n = 0, 5$

But n ≥ 2. so, n = 5.

Q.6 **(C)**



If '2' goes in '1' then it is dearrangement of 4 things which can be done in $4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$ = 9 ways.

If '2' doesn't go in 1, it is dearrangement of 5 things which can be done in 44 ways. Hence total 53 ways.

Q.7 (**5**)

n = 5! × 6!

m = 5! × $^6C_2$ × $^5C_4$ .2! .4!

$$\frac{m}{n} = \frac{5!\times 15\times 2\times 5!}{6!} = 5.$$

Q.8 (A)

If there is no boy then, no. of ways = $^6C_4 \times {}^4C_1$ = 60

If one boy is there, then, no. of ways = $^4C_1 \times {}^6C_3 \times ({}^1C_1 + {}^3C_1)$ = 320

Hence, total no. of ways = 380

Hence, (A)

Q.9 **(C)**

$N_1 = {}^5C_1 \cdot {}_4C_4 = 5$

$N_2 = {}^5C_2 \cdot {}_4C_3 = 40$

$N_3 = {}^5C_3 \cdot {}_4C_2 = 60$

$N_4 = {}^5C_4 \cdot {}_4C_1 = 20$

$N_5 = {}^5C_5 \cdot {}_4C_0 = 1$ Total = 126

Q.10 **(5)**

A, B, C, D, E, F, G, H, I, J

x = 10!

y = $^{10}C_1 . {}^{10}C_2 . 8!\, {}^9C_8$

$$\frac{y}{9x} = \frac{^{10}C_1 . {}^{10}C_2 .8!\times 9}{9\times 10!} = \frac{10!\times 45}{9\times 10!} = 5$$

Q.11 **(625)**

Option for last two digits are

(12), (24), (32), (44) are (52).

∴ Total No. of digits

= 5 × 5 × 5 × 5 = 625

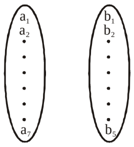
Q.12 **(119)**

n(X) = 5

n(Y) = 7

α → Number of one-one function $^7C_5 \times 5!$

β → Number of onto function Y to X



1, 1, 1, 1, 3 1, 1, 1, 2, 2,

$$\frac{7!}{3!4!}\times 5! + \frac{7}{(2!)^3\,3!}\times 5! = \left({}^7C_3 + 3.{}^7C_3\right)5! = 4\times{}^7C_3\times 5!$$

$$\frac{\beta-\alpha}{5!} = 4\times{}^7C_3 - {}^7C_5 = 4\times 35 - 21 = 119$$

Q.13 **(C)**

(1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$ So P → 4

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1}+\binom{6}{2}\binom{5}{2}+\binom{6}{3}\binom{5}{3}+\binom{6}{4}\binom{5}{4}+\binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1$ = 46!

So Q → 6

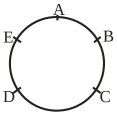
(3) $\alpha_3 = \binom{5}{2}\binom{6}{3}+\binom{5}{3}\binom{6}{2}+\binom{5}{4}\binom{6}{1}+\binom{5}{5}\binom{6}{0} = 381$

So R → 5

(4) $\alpha_2 = \binom{5}{2}\binom{6}{2}-\binom{4}{1}\binom{5}{1}+\binom{5}{3}\binom{6}{1}-\binom{4}{2}\binom{1}{1}+\binom{5}{4} = 189$

So R → 2

Q.14 (30.00)



When 1R, 2B, 2G

$5C_1 \times 2 = 10$

Other possibilities

1B , 2R, 2G

or 1G, 2R, 2B

So total no. of ways = 3 × 10 = 30

Q.15 (495.00)

Selection of 4 days out of 15 days such that no two of them are consecutive

= $^{15-4+1}C_4 = {}^{12}C_4$

$$\frac{12\times 11\times 10\times 9}{4\times 3\times 2} = 11\times 5\times 9 = 495$$

Q.16 (1080.00)

required ways = $\frac{6!}{2!2!1!1!2!2!}\times 4! = 1080$

Binomial Theorem

EXERCISES

ELEMENTRY

Q.1 **(2)**

Applying $T_{r+1} = {}^nC_r x^{n-r} a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5 (2x^2)^5 \left(-\frac{1}{3x^2}\right)^5$

$$= -\frac{10!}{5!5!} 32 \times \frac{1}{243} = -\frac{896}{27}$$

Q.2 **(3)**

${}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow \ 20 - r + 1 = r + 3 \Rightarrow r = 9$.

Q.3 **(3)**

$T_{16} = {}^{17}C_{15} (\sqrt{x})^2 (-\sqrt{y})^{15}$

$= -\frac{17 \times 16}{2 \times 1} \times xy^{15/2} = -136xy^{15/2}$

Q.4 **(3)**

$\frac{1}{6} = \frac{{}^nC_6 (2^{1/3})^{n-6} (3^{-1/3})^6}{{}^nC_{n-6} (2^{1/3})^6 (3^{-1/3})^{n-6}}$ or $6^{-1} = 6^{-4}.6^{n/3} = 6^{n/3-4}$

$\therefore \frac{n}{3} - 4 = -1 \ \Rightarrow \ n = 9.$

Q.5 **(3)**

$T_r = {}^{15}C_{r-1} (x^4)^{16-r} \left(\frac{1}{x^3}\right)^{r-1} = {}^{15}C_{r-1} x^{67-7r}$

$\Rightarrow \ 67 - 7r = 4 \Rightarrow r = 9$.

Q.6 **(2)**

We have $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + ...$

By hypothesis, $\frac{m(m-1)}{2} x^2 = -\frac{1}{8} x^2$

$\Rightarrow \ 4m^2 - 4m = -1 \ \Rightarrow (2m-1)^2 = 0 \Rightarrow m = \frac{1}{2}$.

Q.7 **(2)**

In the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ the general term is

$T_{r+1} = {}^5C_r (x^2)^{5-r} \left(\frac{a}{x}\right)^r = {}^5C_r a^r x^{10-3r}$

Here, exponent of x is $10 - 3r = 1 \Rightarrow r = 3$

$\therefore \ T_{2+1} = {}^5C_3 a^3 x = 10a^3.x$

Hence coefficient of x is $10a^3$.

Q.8 **(1)**

In the expansion of $\left(x - \frac{1}{x}\right)^6$, the general term is

${}^6C_r x^{6-r} \left(-\frac{1}{x}\right)^r = {}^6C_r (-1)^r x^{6-2r}$

For term independent of x, $6 - 2r = 0 \Rightarrow r = 3$

Thus the required coefficient $= (-1)^3.{}^6C_3 = -20$.

Q.9 **(3)**

The coefficient of x^{16} in the expansion of $(x^2 - 2x)^{10}$

= The coefficient of x^{16} in $x^{10}(x-2)^{10}$

= The coefficient of x^6 in $(x-2)^{10}$ f

$= {}^{10}C_4.2^4, \ \left(\because T_{r+1} = {}^nC_r x^{n-r} a^r\right)$

$= 210 \times 16 = 3360$.

Q.10 **(3)**

Let T_{r+1} term containing x^{32}.

Therefore ${}^{15}C_r x^{4r} \left(\frac{-1}{x^3}\right)^{15-r}$

$\Rightarrow \ x^{4r} x^{-45+3r} = x^{32} \Rightarrow 7r = 77 \Rightarrow r = 11$.

Hence coefficient of x^{32} is ${}^{15}C_{11}$ or ${}^{15}C_4$

Q.11 **(2)**

x^7, x^8 will occur in T_8 and T_9.

Coefficients of T_8 and T_9 are equal.

$\therefore {}^nC_7 2^{n-7} \left(\frac{1}{3}\right)^7 = {}^nC_8 2^{n-8} \left(\frac{1}{3}\right)^8 \Rightarrow n = 55$.

Q.12 **(1)**

In the expansion of $\left(\frac{3x^2}{2} + \frac{1}{3x}\right)^9$, the general term is

$T_{r+1} = {}^9C_r . \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For the term independent of x, $18 - 3r = 0 \Rightarrow r = 6$

This gives the independent term

$T_{6+1} = {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \left(-\frac{1}{3}\right)^6 = {}^9C_3 . \frac{1}{6^3}$

Q.13 **(1)**

$T_{r+1} = {}^{10}C_r (x^2)^{10-r} \left(\frac{-3\sqrt{3}}{x^3}\right)^r$

For term independent of x, $20 - 2r - 3r = 0 \Rightarrow r = 4$

$\therefore \ T_{4+1} = {}^{10}C_4 (-3)^4 (\sqrt{3})^4 = 153090.$

Q.14 **(3)**

As in Previous question, obviously the term independent of x will be

${}^nC_0.{}^nC_0 + {}^nC_1.{}^nC_1 + ...{}^nC_n.{}^nC_n = C_0^2 + C_1^2 + ... + C_n^2$.

Q.15 **(2)**

Since n is even therefore $\left(\frac{n}{2}+1\right)^{th}$ term is middle term, hence ${}^{n}C_{n/2}(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2} = 924x^6$

$\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12 \cdot$

Q.16 **(3)**

Middle term = $T_{\frac{2n+2}{2}} = T_{n+1} = {}^{2n}C_n x^n = \frac{2n!}{(n!)^2} \cdot x^n$.

Q.17 **(2)**

Greatest coefficient of $(1+x)^{2n+2}$ is

$$= {}^{(2n+2)}C_{n+1} = \frac{(2n+2)!}{\{(n+1)!\}^2}$$

Q.18 **(2)**

Obviously the middle term

$$= {}^{2n}C_n(x)^n \cdot \left(\frac{1}{2x}\right)^n = \frac{2n!}{n!.n!.2^n} = \frac{1.3.5....(2n-1)}{n!}.$$

Q.19 **(4)**

$$(1+3x+2x^2)^6 = [1+x(3+2x)]^6$$

$$= 1 + {}^6C_1x(3+2x) + {}^6C_2x^2(3+2x)^2$$

$$+ {}^6C_3x^3(3+2x)^3 + {}^6C_4x^4(3+2x)^4$$

$$+ {}^6C_5x^5(3+2x)^5 + {}^6C_6x^6(3+2x)^6$$

Only x^{11} gets from ${}^6C_6x^6(3+2x)^6$

$\because$ ${}^6C_6x^6(3+2x)^6 = x^6(3+2x)^6$

$\therefore$ Coefficient of = .

Q.20 **(3)**

Trick : Put $n =$ 1, 2

At $n = 1, {}^1C_0 - \frac{1}{2}{}^1C_1 = 1 - \frac{1}{2} = \frac{1}{2}$

At $n = 2, {}^2C_0 - \frac{1}{2}{}^2C_1 + \frac{1}{3}{}^2C_2 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$

which is given by option (c).

Q.21 **(3)**

Proceeding as above and putting $n+1=N$.
So given term can be written as

$$\frac{1}{N}\left\{{}^NC_1 + {}^NC_2 + {}^NC_3 + ...\right\}$$

$$= \frac{1}{N}\{2^N - 1\} = \frac{1}{n+1}(2^{n+1} - 1) \qquad (\because N = n+1)$$

Q.22 **(2)**

Multiplying each term by n ! the question reduces to

$$\frac{n!}{1!(n-1)!} + \frac{1}{3!}.\frac{n!}{(n-3)!} + \frac{1}{5!}.\frac{n!}{(n-5)!} + ...$$

$$= {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}.$$

Thus $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... = \frac{1}{n!}2^{n-1}$.

Q.23 **(3)**

Putting x = 1 in $(x^2 + x - 3)^{319}$

We get the sum of coefficient = $(1+1-3)^{319}$ = –1

Q.24 **(3)**

$$(1+x+x^2+x^3)^5 = (1+x)^5(1+x^2)^5$$

$$= (1+5x+10x^2+10x^3+5x^4+x^5)$$

$$\times(1+5x^2+10x^4+10x^6+5x^8+x^{10})$$

Therefore the required sum of coefficients

$$= (1+10+5).2^5 = 16 \times 32 = 512$$

Note : $2^n = 2^5$ = Sum of all the binomial coefficients in the 2nd bracket in which all the powers of x are even.

Q.25 **(3)**

As we know that

$${}^nC_0 - {}^nC_1^2 + {}^nC_2^2 - {}^nC_3^2 + ... + (-1)^n. {}^nC_n^2 = 0,$$

(if n is odd) and in the question n=15 (odd).

Q.26 **(1)**

$$(1.0002)^{3000} = (1 + 0.0002)^{3000}$$

$$= 1 + (3000)(0.0002) + \frac{(3000)(2999)}{1.2}(0.0002)^2 +$$

$$\frac{(3000)(2999)(2998)}{1.2.3}(0.0002)^3 +$$

We want to get answer correct to only one decimal places and as such we have left further expansion.

= $1 + (3000)(0.0002)$ = 1.6

Q.27 **(3)**

We have $7^2 = 49 = 50 - 1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

$$= {}^{150}C_0(50)^{150}(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + ...$$

$$+ {}^{150}C_{150}(50)^0(-1)^{150}$$

Thus the last digits of 7^{300} are ${}^{150}C_{150}.1.1$ *i.e.*, 1.

Q.28 **(1)**

111…..1 (91 times)

$= 1 + 10 + 10^2 + ... + 10^{90}$

$= \frac{10^{91}-1}{10-1} = \frac{(10^7)^{13}-1}{10-1} = \frac{t^{13}-1}{9}$, where $t = 10^7$

$$= \left(\frac{t-1}{9}\right)(t^{12} + t^{11} + ... + t + 1)$$

$$= \left(\frac{10^7-1}{10-1}\right)(1 + t + t^2 + ... + t^{12})$$

$$= (1 + 10 + 10^2 + ... + 10^6)(1 + t + t^2 + ... + t^{12})$$

$\therefore$ $111.....1(91\ times)$ is a composite number.

Q.29 **(1)**

Given term can be written as $(1+x)^2(1-x)^{-2}$

$$=(1+2x+x^2)[1+2x+3x^2+....+(n-1)x^{n-2}$$

$$+nx^{n-1}+(n+1)x^n+....]$$

$$=x^n(n+1+2n+n-1)+....$$

Therefore coefficient of x^n is 4*n*.

Q.30 **(2)**

Let consecutive terms are ${}^nC_r$ and ${}^nC_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!r!}=\frac{n!}{(n-r-1)!(r+1)!}$$

$$\Rightarrow \frac{1}{(n-r)(n-r-1)!r!}=\frac{1}{(n-r-1)!(r+1)r!}$$

$\Rightarrow r+1=n-r \Rightarrow n=2r+1$. Hence *n* is odd.

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 **(3)**

$${}^{2m+1}C_m \left(\frac{x}{y}\right)^{m+1}\left(\frac{y}{x}\right)^{m} = {}^{2m+1}C_m\left(\frac{x}{y}\right)$$

Dependent upon the ratio $\frac{x}{y}$ and m.

Q.2 **(3)**

$(x + a)^{100} + (x - a)^{100}$

= 2

$\left({}^{100}C_0\, x^{100} + {}^{100}C_2\, x^{98} a^2 + ... + {}^{100}C_{100}\, a^{100}\right)$

Number of terms = 51 terms

Q.3 **(1)**

$$T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2\log_{10} x)^5 = 5600$$

$$\Rightarrow \frac{1}{x^8}\ x^{10}(\log_{10} x)^5 = 100$$

$\Rightarrow x = 10$

Q.4 **(2)**

$$T_{11} = {}^{15}C_{10}\ (3)^5\left(-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{10}$$

positive irrational number

Q.5 **(1)**

$T_2 = {}^nC_1\,(a^{1/13})^{n-1}\,(a^{3/2}) = 14a^{5/2}$

$\Rightarrow n = 14$

$$\therefore\ \frac{{}^nC_3}{{}^nC_2} = 4$$

Q.6 **(1)**

$(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^r = ({}^nC_0x^0 + {}^nC_2x^2 + ……) + ({}^nC_1x^1 + {}^nC_3x^3 + ……) = (b + a)$

$$\begin{cases} T_2+T_4+T_6+……=b \\ T_1+T_3+T_5+……=a \end{cases}$$

$\therefore\ (1 - x)^n = ({}^nC_0x^0 + {}^nC_2x^2 + ……) - ({}^nC_1x^1 + {}^nC_3x^3 + ……) = b - a$

$(1 - x^2)^n = (1 + x)^n (1 - x)^n = (a + b) (a - b) = a^2 - b^2$

Q.7 **(3)**

$$\left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r\left(x^{\frac{1}{3}}\right)^{15-r}\left(-x^{-\frac{1}{2}}\right)^r$$

$$T_{r+1} = {}^{15}C_r(-1)^r\ (x)^{\frac{15-r}{3}-\frac{r}{2}} = {}^{15}C_r(-1)^r\ (x)^{\frac{30-5r}{6}}$$

Given if $\frac{30-5r}{6} = 0$ then $T_{r+1} = 5m, m \in N$

$\Rightarrow\ r = 6 \Rightarrow T_7 = 5m$

$$T_7 = {}^{15}C_6\ (-1)^6 = \frac{15\cdot14\cdot13\cdot12\cdot11\cdot10}{6\cdot5\cdot4\cdot3\cdot2}$$

$= 5\cdot7\cdot13\cdot11\cdot = 5\cdot(1001) \Rightarrow m = 1001$

Q.8 **(2)**

7th term of $\left(2^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)^n$ is $T_7 = {}^nC_6\, 2^{\frac{n-6}{3}}\, 3^{-2}$

From begining

7th term of $\left(2^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)^n$ from the end

= 7th term of $\left(3^{-\frac{1}{3}}+2^{\frac{1}{3}}\right)^n$ from the begining

$$=T_7' = {}^nC_6\ 3^{-\left(\frac{n-6}{3}\right)}\ 2^2$$

given that $\frac{T_7}{T_7'} = \frac{1}{6}$

$$\Rightarrow \frac{{}^nC_6\ 2^{\frac{n-6}{3}}\ 3^{-2}}{{}^nC_6\ 3^{-\left(\frac{n-6}{3}\right)}\ 2^2} = \frac{1}{6} \qquad \Rightarrow \frac{2^{\frac{n-6}{3}-2}}{3^{2-\left(\frac{n-6}{3}\right)}} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{(2.3)^{2-\left(\frac{n-6}{3}\right)}} = \frac{1}{6} \Rightarrow\ 2-\frac{n-6}{3} = 1$$

$\Rightarrow\ 3 = n - 6 \Rightarrow\ n = 9$

Q.9 **(3)**

$$(1 - 2x^3 + 3x^5)\left(1+\frac{1}{x}\right)^8$$

Co-efficient of x = $-2.\,{}^{8}C_2 + 3.{}^{8}C_4 = 154$

Q.10 **(1)**

$$T_{r+1} = {}^{6561}C_r\left(7\right)^{\frac{6561-r}{3}}(11^{1/9})^r$$

Here r should be multiple of 9

r = 0, 9,18...............6561

Number of terms = 730

Q.11 **(2)**

$$\left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)^3 = \left(x-\frac{1}{x}\right)({}^{3}C_0\,x^6 - {}^{3}C_1\,x^2 + {}^{3}C_2\,x^{-2} - {}^{3}C_3\,x^{-6})$$

There is no term independent of x

Q.12 (**2**)

$$\left.\begin{matrix} T_{2m+1} \Rightarrow {}^{10}C_{2m} \\ T_{4m+5} \Rightarrow {}^{10}C_{4m+4} \end{matrix}\right] \text{ equal}$$

2m + 4m + 4 = 10; 6m + 4 = 10

m = 1

Q.13 (**3**)

$$T_{r+1} = {}^{n}C_r(2)^{n-r}\left(\frac{x}{3}\right)^r = {}^{n}C_r\,\frac{2^{n-r}}{3^r}x^r$$

Coeff $T_8 = T_9 \Rightarrow {}^{n}C_7\,\frac{2^{n-7}}{3^7} = {}^{n}C_8\,\frac{2^{n-8}}{3^8}$

$$\Rightarrow \frac{{}^{n}C_8}{{}^{n}C_7} = \frac{3.2}{1} \Rightarrow \frac{n-8+1}{8} = 6$$

$\Rightarrow$ n – 7 = 48 $\Rightarrow$ n = 55

Q.14 (**4**)

$$\frac{1}{\sqrt{4x+1}}\left[\left(\frac{1+\sqrt{4x+1}}{2}\right)^7 - \left(\frac{1-\sqrt{4x+1}}{2}\right)^7\right]$$

$$= \frac{1}{2^7\sqrt{4x+1}}\left[\sum_{r=0}^{7}{}^{7}C_r\left(1\right)^{7-r}\left(\sqrt{4x+1}\right)^r\right.$$

$$\left.-\sum_{r=0}^{7}{}^{7}C_r(1)^{7-r}\left(-\sqrt{4x+1}\right)^r\right]$$

$$= \frac{1}{2^7\sqrt{4x+1}}\,[T_1+T_2+T_3+...+T_8-T_1+T_2-T_3+...+T_8]$$

$$= \frac{1}{2^7\sqrt{4x+1}}\left[2\left(T_2+T_4+T_6+T_8\right)\right]$$

$$= \frac{1}{2^6\sqrt{4x+1}}\left[{}^{7}C_1\sqrt{4x+1} + {}^{7}C_3\left(\sqrt{4x+1}\right)^3\right.$$

$$\left.+\,{}^{7}C_5\left(\sqrt{4x+1}\right)^5 + {}^{7}C_7\left(\sqrt{4x+1}\right)^7\right]$$

$$= \frac{1}{2^6}\left[{}^{7}C_1 + {}^{7}C_3\left(4x+1\right)\right.$$

$$\left.+\,{}^{7}C_5\left(4x+1\right)^2 + {}^{7}C_7\left(4x+1\right)^3\right]$$

= is the polynomial in x of degree 3

Q.15 (**2**)

G.T. is $T_{r+1} = {}^{100}C_r(2)^{\frac{100-r}{2}}(3)^{\frac{r}{4}}$

The above term will be rational if exponent of 2 & 3 are integers.

i.e. $\frac{100-r}{2}$ and $\frac{r}{4}$ must be integers

the possible set of r is = {0, 4, 8, 16,……, 100}

no. of rational terms is 26

Q.16 (**2**)

If $n \in N$ & n is even then

$$\frac{1}{1\cdot(n-1)!} + \frac{1}{3!(n-3)} + \frac{1}{5!(n-5)} + \ldots + \frac{1}{(n-1)!\ 1!}$$

$$= \frac{1}{n!}\left[{}^{n}C_1 + {}^{n}C_3 + {}^{n}C_5 + \ldots\ldots + {}^{n}C_{n-1}\right]$$

n is even $\Rightarrow$ n – 1 is odd

${}^{n}C_{n-1}$ second Binomail coeff. from the end

$$= \frac{1}{n!}\left[C_1 + C_3 + C_5 + \ldots\ldots + C_{n-1}\right]$$

$$= \frac{1}{n!}\cdot 2^{n-1} = \frac{2^{n-1}}{n!}$$

Q.17 (**2**)

middle term = T_5

$T_5 = T_{4+1} = {}^{8}C_4\,.\,k^4 = 1120$

$\Rightarrow$ k = 2

Q.18 (**1**)

$C_0 + C_1 + C_2 + \ldots\ldots + C_n = 2^n = 256$

$\Rightarrow 2^n = 2^8 \Rightarrow n = 8$

$$T_{r+1} = {}^{8}C_r\left(2x\right)^{8-r}\left(\frac{1}{x}\right)^r = {}^{8}C_r\,2^{8-r}x^{8-2r}$$

For Constant term $\Rightarrow$ 8 – 2r = 0 $\Rightarrow$ r = 4

$$= {}^{8}C_4\,2^4 = \frac{8\times7\times6\times5}{4\times3\times2}2^4 = 70\times16 = 1120$$

Q.19 (**1**)

sum of coeff of $(1-2x+5x^2)^n = a$

sum of coeff of $(1+x)^{2n} = b$

put x = y = 1

$a = (1-2+5)^n = 4^n$ & $b = (1+1)^{2n} = 2^{2n} = 4^n$

a = b

Q.20 (**4**)

$\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ Independent of x

$$T_{r+1} = {}^{3n}C_r\left(x^k\right)^{3n-r}\left(\frac{1}{x^{2k}}\right)^r$$

$= {}^{3n}C_r\, x^{3nk - rk - 2kr} = {}^{3n}C_r\, x^{3k(n - r)}$

For Constant term $\Rightarrow$ $3k(n - r) = 0 \Rightarrow n = r$

$\therefore T_{r+1} = {}^{3n}C_n$ true for any real k or $K \in R$

Q.21 **(1)**

$(3x + 2)^{-1/2}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

Q.22 **(2)**

Coeff of α^t in

$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2$

$\ldots\ldots + (\alpha + q)^{m-1}$

$\because\ a \neq -q,\ p \neq q$

Let $\alpha + P = x$ & $\alpha + q = y$

$= x^{m-1} + x^{m-2}y + x^{m-3}y^2 + \ldots\ldots y^{m-1}$

$$= x^{m-1}\left[1 - \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 + \ldots\ldots + \left(\frac{y}{x}\right)^{m-1}\right]$$

$$= x^{m-1}\frac{\left[1 - \left(\frac{y}{x}\right)^m\right]}{\left(1 - \frac{y}{x}\right)}$$

$$= \frac{x^{m-1}}{x^m}\frac{x^m - y^m}{x - y}\cdot x = \frac{(\alpha + p)^m - (\alpha + q)^m}{\alpha + p - \alpha - q}$$

$$= \frac{1}{(p - q)}\left[(\alpha + p)^m - (\alpha + q)^m\right]$$

$$= \text{coeff of } \alpha^t = \left(\frac{{}^mC_t\, p^{m-t} - {}^mC_t\, q^{m-t}}{p - q}\right)$$

Q.23 **(3)**

$(2x + 5y)^{13}$ greatest form for $x = 10$, $y = 2$

$$\frac{n+1}{\left|\frac{x}{y}\right| + 1} - 1 - r - \frac{n+1}{\left|\frac{x}{y}\right| + 1}$$

$$\Rightarrow \frac{14}{\left|\frac{2x}{5y}\right| + 1} - 1 - r - \frac{14}{\left|\frac{2x}{5y}\right| + 1}$$

$$\Rightarrow \frac{14}{3} - 1 \,,,\, r \,,,\, \frac{14}{3} \Rightarrow \frac{11}{3} \leqslant r \leqslant \frac{14}{3}$$

$\Rightarrow 3.66 \ldots\ldots \square\, r \leqslant 4.666 \Rightarrow r = 4$

$\Rightarrow T_5 = {}^{13}C_4\,(20)^9\,(10)^4$

Q.24 **(1)**

For numerically greatest term

$$r = \left[\frac{n+1}{1 + \left|\frac{x}{a}\right|}\right] = \left[\frac{9+1}{1 + \left|\frac{4}{9}\right|}\right] \quad \Rightarrow r = 6$$

Numerically greatest term $T_{r+1} = {}^9C_6\,(2)^3\left(\frac{9}{2}\right)^6$

Q.25 **(2)**

$$\text{For numerically greatest term } r = \left[\frac{n+1}{1 + \left|\frac{x}{a}\right|}\right] = \left[\frac{34+1}{1 + \left|\frac{6}{10}\right|}\right]$$

$\Rightarrow r = 21.$

Q.26 **(1)**

Sol. $$\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \sum_{r=0}^{n-1}\frac{r+1}{n+1}$$

$$= \frac{1}{n+1}[1 + 2 + \ldots\ldots\ldots + n] = \frac{1}{n+1} \times \frac{n(n+1)}{2} = \frac{n}{2}$$

Q.27 **(3)**

${}^{18}C_{r-2} + 2\cdot{}^{18}C_{r-1} + {}^{18}C_r\ \square\ {}^{20}C_{13}$

$\Rightarrow {}^{18}C_{r-2} + {}^{18}C_{r-1} + {}^{18}C_{r-1} + {}^{18}C_r\ \square\ {}^{20}C_{13}$

$\Rightarrow {}^{19}C_{r-1} + {}^{19}C_r\ \square\ {}^{20}C_{13} \Rightarrow {}^{20}C_r\ \square\ {}^{20}C_{13}$

$\therefore {}^{20}C_{10} > {}^{20}C_{11} > {}^{20}C_{12} > {}^{20}C_{13}$

& ${}^{20}C_{10} > {}^{20}C_9 > {}^{20}C_8 > {}^{20}C_7$

$r = 7, 8, 9, 10, 11, 12, 13 \Rightarrow$ Total 7 elements

Q.28 **(3)**

$$\left(\sum_{r=0}^{10}{}^{10}C_r\right)\left(\sum_{k=0}^{10}(-1)^k\frac{{}^{10}C_k}{2^k}\right)$$

$= ({}^{10}C_0 + \ldots\ldots\ldots +$

$${}^{10}C_{10})\left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2}\ldots\ldots\ldots + \frac{{}^{10}C_{10}}{2^{10}}\right)$$

$$= 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$$

Q.29 **(2)**

$$\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots + \frac{C_{10}}{11}$$

$$\sum_{r=0}^{10}T_{r+1} = \sum_{r=0}^{10}\frac{{}^{10}C_r}{r+1}$$

$$= \sum_{r=0}^{10}\frac{1}{(10+1)}\cdot\frac{10+1}{r+1}\cdot{}^{10}C_r = \frac{1}{11}\sum_{r=0}^{10}{}^{11}C_{r+1}$$

$$= \frac{1}{11}\left[{}^{11}C_1 + {}^{11}C_2 + \ldots\ldots + {}^{11}C_{11}\right]$$

$$= \frac{1}{11}\left[{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \ldots + {}^{11}C_{11} - {}^{11}C_0\right]$$

$$= \frac{1}{11}\left[2^{11} - 1\right] = \frac{2^{11}-1}{11}$$

Q.30 **(2)**

$$\frac{{}^{11}C_0}{1} + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \ldots\ldots + \frac{{}^{11}C_{10}}{11}$$

$$= \frac{1}{12}$$

$$\left[\frac{12}{1}\cdot{}^{11}C_0 + \frac{12}{2}\cdot{}^{11}C_1 + \frac{12}{3}\cdot{}^{11}C_2 + \ldots\ldots + \frac{12}{11}\cdot{}^{11}C_{10}\right]$$

$$= \frac{1}{12}\left[{}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \ldots\ldots\ldots + {}^{12}C_{11}\right]$$

$$= \frac{1}{12}\ (2^{12} - 2) = \frac{2^{11}-1}{6}$$

Q.31 **(3)**

$$\int_0^1 (1-x)^n\,dx = \int_0^1 \left(C_0 - C_1x + C_2x^2 - C_3x^3 + \ldots + (-1)^n C_nx^n\right)dx$$

$$\Rightarrow \frac{1}{n+1} =$$

$$\left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots\ldots\ldots + (-1)^n\frac{C_n}{n+1}\right]$$

$$\Rightarrow \frac{1}{3}\left(\frac{1}{n+1}\right) =$$

$$\frac{1}{3}\left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + (-1)^n\frac{C_n}{n+1}\right]$$

Q.32 **(3)**

${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{52}C_4$

Q.33 **(2)**

${}^{50}C_0 \times {}^{50}C_1 + {}^{50}C_1 \times {}^{50}C_2 + \ldots\ldots\ldots + {}^{50}C_{49} \times {}^{50}C_{50}$
$= {}^{50}C_0 \times {}^{50}C_{49} + {}^{50}C_1 \times {}^{50}C_{48} + \ldots\ldots\ldots + {}^{50}C_{49} \times {}^{50}C_0$
= co-eff. of x^{49} in $(1 + x)^{100} = {}^{100}C_{49}$

Q.34 **(1)**

$(1 + x + x^2)^{25} = a_0 + a_1x + a_2x^2 + \ldots\ldots a_{50}x^{50}$
put x = 1
$a_0 + a_1 + a_2 + a_3 + \ldots\ldots + a_{49} + a_{50} = 3^{25}$
…(i)
put x = –1
$a_0 - a_1 + a_2 - a_3 + \ldots + a_{49} + a_{50} = (1 - 1 + 1)^{25}$…(ii)
adding (i) & (ii)
$2[a_0 + a_2 + a_4 + \ldots\ldots + a_{50}] = 3^{25} + 1$

$$a_0 + a_2 + a_4 + \ldots\ldots + a_{50} = \frac{3^{25}+1}{2} = \frac{(4-1)^{25}+1}{2}$$

$= {}^{25}C_0\,4^{25} - {}^{25}C_1\,4^{24} + {}^{25}C_2\,4^{23} - \ldots\ldots + {}^{25}C_{24}\,4^1 - 1$
$+ 1$

$$= \frac{4\left[{}^{25}C_0 4^{24} - {}^{25}C_1 4^{23} + \ldots\ldots\ {}^{25}C_{24}\right]}{2}$$

Q.35 **(2)**

$$\sum_{k=1}^{n-r} {}^{n-k}C_r = {}^{x}C_y$$

L.H.S. $= {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \ldots\ldots + {}^{r}C_r$
$= {}^{r}C_r + {}^{r+1}C_r + \ldots\ldots\ {}^{n-2}C_r + {}^{n-1}C_r$

$$\left\{{}^{r}C_r = \frac{r+1}{r+1}{}^{r}C_r = {}^{r+1}C_{r+1}\right\}$$

$= {}^{r+1}C_{r+1} + {}^{r+1}C_r + {}^{r+2}C_r + \ldots\ldots + {}^{n-1}C_r$
$= {}^{r+2}C_{r+1} + {}^{r+2}C_r + \ldots\ldots + {}^{n-1}C_r$
$= {}^{r+2}C_{r+1} + \ldots\ldots + {}^{n-1}C_r$
$= {}^{n-1}C_{r+1} + {}^{n-1}C_r$
$= {}^{n}C_{r+1} = {}^{x}C_y \Rightarrow x = n, y = r + 1$

Q.36 **(3)**

$2^{2003} = 8.(16)^{500}$
$= 8\ (17–1)^{500}$
∴ Remainder = 8

Q.37 **(4)**

$3^{400} = (10–1)^{200}$
${}^{200}C_0 (10)^{200} + \ldots\ldots.. + {}^{200}C_{199} (10)\ (–1) + {}^{200}C_{200}$
Last two digits = 01

Q.38 **(1)**

Last two digits in 10! are 00 and third digit = 8

Q.39 **(1)**

$$\sum_{r=1}^{10} r.\frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \sum_{r=1}^{10} n - r + 1$$

$$= (n + 1) \times 10 - \frac{10\times11}{2}$$

= 10n – 45

Q.40 **(2)**

T_{22} is the numerically greatest term.

$(\sqrt{2} + 1)^6 = I + f$

$(\sqrt{2} - 1)^6 = f'$

____

$2[{}^6C_0 + {}^6C_2\ .\ 2 + {}^6C_4\ (2)^2 + \ldots\ldots..] = I + f + f'$
$f + f' = 1$ or $f' = 1 – f$
$I = 2\ [{}^6C_0 + {}^6C_2 .2 + {}^6C_4 .4 + {}^6C_6.8] –1$
$I = 2\ [1 + 30 + 60 + 8] – 1 = 197$

Q.41 **(2)**

$$\left(5+2\sqrt{6}\right)^n = p + f$$

$$\left(5-2\sqrt{6}\right)^n = f' \Rightarrow 0 < f + f' < 2$$

p + f + f ' = 2 [integer]

so f + f ' = integer = 1

$\because n \in N$

(f – 1)(p + f) = –f ' (p + f) = –(+1)^n = –1

Q.42 **(3)**

$3^{1/3}$	$7^{1/7}$	1
0	0	10
3	0	7
6	0	4
9	0	1
3	7	0
0	7	3

$\therefore$ no. of terms are 6

Alter : $^{10}C_r\left(1+3^{\frac{1}{3}}\right)^{10-0} \Rightarrow {}^{10}C_r\left(1+3^{\frac{1}{3}}\right)^{r}$

should be r = 0, 3,6, 9

$\left(1+3^{\frac{1}{3}}\right)^{10-7} \Rightarrow {}^{10}C_r\left(3^{\frac{1}{3}}\right)^{r}$ r shoule be r = 0, 3

corresponding $\left(7^{\frac{1}{7}}\right)^{0} \left(3^{\frac{1}{3}}\right)^{r}$

value of r = 0, 3, 6, 9 = 4 values

corresponding $\left(7^{\frac{1}{7}}\right)^{0} \left(3^{\frac{1}{3}}\right)^{r}$

value of r = 0, 3 = 2 values

Total 6 values

Q.43 **(4)**

Co-efficient of x^n in $(1 - x)^{-2} = {}^{2+n-1}C_1 = n + 1$

Q.44 **(4)**

coef of x^4 in $(1 - x + 2x^2)^{12}$

$= {}^{12}C_0 (1 - x)^{12} (2x^2)^0 + {}^{12}C_1 (1 - x)^{11} (2x^2) + {}^{12}C_2 (1 - x)^{10} (2x^2)^2$ + above x^4 powers terms of x^4

$= {}^{12}C_0 \cdot {}^{12}C_4 (-x)^4 + {}^{12}C_1\, {}^{11}C_2 (-x)^2\, 2x^2 + {}^{12}C_2\, {}^{10}C_0\, 4x^4$

$= {}^{12}C_4 + 12 \cdot {}^{11}C_2 \cdot 2 + {}^{12}C_2 \cdot 4$

$= {}^{12}C_4 + 2.3 \cdot \frac{12}{3}\, {}^{11}C_2 + {}^{12}C_2 \cdot 4$

$$= {}^{12}C_3 + {}^{12}C_2 + 3\left({}^{12}C_2 + {}^{12}C_3\right) + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$

$$= {}^{12}C_3 + 3({}^{12}C_2 + {}^{12}C_3) + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$

$$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{13}C_3 + {}^{13}C_4$$

$$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{14}C_4$$

Q.45 **(4)**

$(1 + x)^2 (1 - x)^{-2}$

$= (1 + x^2 + 2x) (1 - x)^{-2}$

Co-efficient of $x^4 = {}^5C_4 + {}^3C_2 + 2\, {}^4C_3 = 16$

Q.46 **(1)**

$(1 + x)^{10} = a_0 + a_1 + a_2x^2 + ... + a_{10}x^{10}$

Put x = i,

$(1 + i)^{10} = a_0 - a_2 + a_4 + ...+ a_{10} + i\,(a_1 - a_3 + ...+ a_9)$

$a_0 - a_2 + a_4 + ...+ a_{10}$ = real part of $(1 + i)^{10} = 2^5\cos 10\pi/4$

$a_1 - a_3 + ...$ = imaginary part of $(1 + i)^{10} = 2^5 \sin 10\pi/4$ ....(2)

$(1)^2 + (2)^2 = 2^{10}$

Q.47 **(3)**

Sum of the coeff of degree r is

$(1 + x)^n (1 + y)^n (1 + z)^n$

$$= \left(\sum_{f=0}^{n} {}^nC_k x^k\right)\left(\sum_{s=0}^{n} {}^nC_s y^s\right)\left(\sum_{t=0}^{n} {}^nC_t z^t\right)$$

$$= \sum_{0\le k,s,t\le n} \left({}^nC_k\right)\left({}^nC_s\right)\left({}^nC_t\right)x^k y^s z^t$$

degree m = k + s + t = r

sum of coeff $= \sum_{k,s,t\ge 0} {}^nC_k \cdot {}^nC_s \cdot {}^nC_t$

= the number of way of choosing a total number r balls out of n white, n block and n red balls.

$= {}^{3n}C_r$

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 **(C)**

$$\frac{1}{1!(n-1)!} + \frac{1}{2!(n-2)!} + ... + \frac{1}{1!(n-1)!} = \frac{1}{n!}$$

$[{}^nC_1 + {}^nC_2 + ... + {}^nC_{n-1}]$ (multiply and divde by n!)

$= \frac{1}{n!}\,[2^n - 2] = \frac{2}{n!}\,(2^{n-1} - 1)$

Q.2 **(C)**

$(1 + x)^{21} [1 + (1 + x) + ... (1 + x)^9] = (1 + x)^{21}$

$$\left[\frac{(1+x)^{10} - 1}{x}\right] = \frac{(1+x)^{31} - (1+x)^{21}}{x}$$

Coefficient of $x^5 = {}^{31}C_6 - {}^{21}C_6$

Q.3 **(C)**

$$\left(x^3 - \frac{1}{x^2}\right)^n$$

General term $= \frac{n!}{r!(n-r)!} (-1)^{n-r}\, x^{5r-2n}$

If 5r – 2n = 5, then $5r = 2n + 5 \Rightarrow r = \frac{2n}{5} + 1$

If 5r – 2n = 10, then 5r = 2n + 10 ⇒ $r = \frac{2n}{5} + 2$

Let n = 5k

Now $\frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0$

$\Rightarrow \frac{1}{3k-1} - \frac{1}{2k+2} = 0$

⇒ k = 3 ⇒ n = 15

Q.4 **(D)**

$$\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}}\right)^{10}$$

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r}\left(-\frac{1}{\sqrt{x}}\right)^r$

For independent term $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$

Coefficient of the term independent of x = ${}^{10}C_4$

Q.5 **(C)**

$(x + 3)^n + (x + 3)^{n-1}(x + 2) + \ldots\ldots + (x + 2)^n = (x + 3)^n \left(\frac{1 - \left(\frac{x+2}{x+3}\right)^{n+1}}{1 - \frac{x+2}{x+3}}\right) = [(x + 3)^{n+1} - (x + 2)^{n+1}]$

Coefficient of $x^{n-1} = {}^{n+1}C_{n-1}(3)^2 - {}^{n+1}C_{n-1} \times 4$

Q.6 **(B)**

$(x + 2)^{n-1} + (x + 2)^{n-2}(x + 1) + (x + 2)^{n-3}(x + 1)^2 + \ldots\ldots + (x + 1)^{n-1}$

$= a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots\ldots + b^{n-1}$

$$= a^{n-1}\left[1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 + \ldots\ldots + \left(\frac{b}{a}\right)^{n-1}\right]$$

$$= a^{n-1}\frac{1 \cdot \left[\left(\frac{b}{a}\right)^n - 1\right]}{\frac{b}{a} - 1} = a^{n-1}\frac{a^n - b^n}{a - b} \cdot \frac{a}{a^n}$$

$$= \frac{(x+2)^n - (x+1)^n}{x + 2 - x - 1} = (2 + x)^n - (1 + x)^n$$

T_{r+1} term is ${}^nC_r 2^{n-r} x^r - {}^nC_r x^r$

coeff of x^r is ${}^nC_r(2^{n-r} - 1)$

Q.7 **(D)**

$${}^4C_1 k^3\left(\frac{1}{k}\right) + {}^4C_2 \cdot k^2 \cdot \left(\frac{1}{k^2}\right) + {}^4C_3 \cdot k \cdot \left(\frac{1}{k^3}\right) = 23$$

$\Rightarrow 4k^2 + 6 + 4\left(\frac{1}{k^2}\right) = 23 \Rightarrow 4k^4 + 4 = 17k^2$

$\Rightarrow 4k^4 - 16k^2 - k^2 + 4 = 0$

$\Rightarrow (4k^2 - 1)(k^2 - 4) = 0 \Rightarrow k = \pm\frac{1}{2}, \pm 2.$

Q.8 **(B)**

One middle term

⇒ n = even

x = 3; a = 2; 7th term

${}^nC_5 \cdot 3^{n-5} \cdot 2^5 < {}^nC_6 \cdot 3^{n-6} \cdot 2^6 > {}^nC_7 \cdot 3^{n-7} \cdot 2^7$

$\Rightarrow \quad \frac{3}{2} < \frac{n!}{6!(n-6)!} \times \frac{5!(n-5)!}{n!}$

$\frac{3}{2} > \frac{n!}{7!(n-7)!} \times \frac{6!(n-6)!}{n!}$

$\Rightarrow \quad 2(n - 5) > 3 \cdot 6$

$\Rightarrow \quad 2(n - 6) < 21$

$\Rightarrow \quad n - 5 > 9$

$\Rightarrow \quad 2n < 33$

$\Rightarrow \quad n > 14$

$\Rightarrow \quad n < 16 \cdot 5$

and n is even, so n = 16

$n = 2^4$

Number of divisors = 4 + 1 = 5

Q.9 **(C)**

$$r = \frac{n+1}{1 + \left|\frac{x}{y}\right|} = \frac{14+1}{1 + \left|\frac{3}{2}\right|} = \frac{15 \cdot 2}{5} = 6$$

T_6 and T_7 have numerically greatest coefficients

Coefficient of T_6 is negative and coefficient of T_7 is positive

∴ T_6 has algebraically least coefficient.]

Q.10 **(B)**

$$S = \sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$$

$S = {}^{100}C_0 (x - 3)^{100} + {}^{100}C_1 (x - 3)^{99} . 2 + \ldots\ldots + {}^{100}C_{100}$

. 2^{100}

$S = (2 + (x - 3))^{100} = (x - 1)^{100}$

Co-efficient of $x^{52} = {}^{100}C_{52} = {}^{100}C_{48}$

Q.11 **(B)**

$(x^4 - 1)^5 (x - 1)^{-5}$

$= {}^5C_0 (x - 1)^{-5} - {}^5C_1 x^4 (x - 1)^{-5} + {}^5C_2 x^8 (x - 1)^{-5}$

$= {}^5C_0 \times {}^{14}C_4 - {}^5C_1 \times {}^{10}C_6 + {}^5C_2 \times {}^6C_2 = 101$

Q.12 **(A)**

$$\left(1+\frac{1}{4n}\right)^{4n} = {}^{4n}C_0\left(\frac{1}{4n}\right)^1 + {}^{4n}C_2\left(\frac{1}{4n}\right)^2 + \ldots\ldots + {}^{4n}C_{4n}\left(\frac{1}{4n}\right)^{4n}$$

$$= 1+1+\frac{1}{2!}^{\left(1+\frac{1}{4n}\right)} + \ldots\ldots + \frac{1}{4n!}^{\left(1-\frac{1}{4n}\right)\left(1-\frac{2}{4n}\right)\ldots\ldots\left(1-\frac{4n-1}{4n}\right)}$$

$$< 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots\ldots\frac{1}{(4n)!}$$

$$\left(1+\frac{1}{3n}\right)^{3n} = 1+1+\frac{1}{2!}^{\left(1-\frac{1}{3n}\right)} + +\frac{1}{(3n)!}^{\left(1-\frac{1}{3n}\right)\left(1-\frac{2}{3n}\right)\ldots\ldots\left(1-\frac{3n-1}{3n}\right)}\ldots\ldots$$

$$< 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots\ldots+\frac{1}{(3n)!}$$

$$\left(1+\frac{1}{2n}\right)^{2n} = 1+1+\frac{1}{2!}^{\left(1-\frac{1}{2n}\right)} + \ldots\ldots$$

$$+\frac{1}{(2n)!}^{\left(1-\frac{1}{2n}\right)\left(1-\frac{2}{2n}\right)\ldots\ldots\left(1-\frac{2n-1}{2n}\right)}$$

$$< 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots\ldots+\frac{1}{(2n)!}$$

$$\left(1+\frac{1}{n}\right)^{n} = 1+1+\frac{1}{2!}^{\left(1-\frac{1}{n}\right)} + \ldots\ldots$$

$$+\frac{1}{(n)!}^{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\left(1-\frac{n-1}{n}\right)}$$

$$< 1+1+\frac{1}{2!}+\frac{1}{3!}+\ldots\ldots+\frac{1}{(2n)!}$$

$$\left(1+\frac{1}{n}\right)^{n} < \left(1+\frac{1}{2n}\right)^{2n} < \left(1+\frac{1}{3n}\right)^{3n} < \left(1+\frac{1}{4n}\right)^{4n}$$

Q.13 **(B)**

$(1^2 + 1)1! + (2^2 + 1)2! + (3^2 + 1)3! + \ldots\ldots + (n^2 + 1)\cdot n!$

$$T_n = \sum_{n=1}^{n} (n^2 + 1)n!$$

$$= \sum_{n=1}^{n} \left[n^2 + 3n + 2 - 3n - 1\right]n!$$

$$= \sum_{n=1}^{n} \left[(n+2)(n+1) - 3(n+1) + 2\right]n!$$

$$= \sum_{n=1}^{n} \left[(n+2)\ \ ! \ - 3(n+1)! + 2n!\right]$$

$= 3! + 4! + 5! + \ldots\ldots + n! + (n + 1)! + (n + 2)!$

$- 3[2! + 3! + 4! + 5! + \ldots\ldots + n! + (n + 1)!]$

$+ 2[1! + 2! + 3! + \ldots\ldots + n!]$

$= [(n + 2)! + (n + 1)! - 3(n + 1)!$

$- 3.2! + 2.1! + 2.2!]$

$= (n + 2)! - 2(n + 1)! - 2! + 2.1!$

$= (n + 1)! [n + 2 - 2]$

$= (n + 1)! \cdot n$

$= n \cdot (n + 1)!$

Q.14 **(A)**

$\because \quad (1+x)^n = C_0 + C_1x + \ldots\ldots\ldots\ldots + C_nx^n$

Multiply by x & then differentiate

$(1+x)^n + x.\ n(1+x)^{n-1} = C_0 + 2C_1x + \ldots\ldots\ldots\ldots + (n+1)C_nx^n$

.................(i)

and $(x+1)^n = C_0x^n + C_1x^{n-1} + \ldots\ldots\ldots\ldots + C_n$ ....(ii)

Multiply (i) & (ii) & equate the coefficient of x^n on both side

$C_0^2 + 2C_1^2 + \ldots\ldots\ldots + (n + 1)\ C_n^2 = {}^{2n}C_n + n.\ {}^{2n-1}C_{n-1}$

$$= \frac{(2n)!}{(n!)^2} + n\frac{(2n-1)!}{n!(n-1)!} = (n+2)\frac{(2n-1)!}{n!(n-1)!}$$

Q.15 **(D)**

$$a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$$

$$S = \sum_{r=0}^{n} \frac{n-2r}{{}^nC_r}$$

$$S = \sum_{r=0}^{n} \frac{n-2(n-r)}{{}^nC_r}$$

$2S = 0 \Rightarrow S = 0$

Q.16 **(B)**

Let $(2x^2 - 3x + 1) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + a_{22}x^{22}$ … (1)

Put x = –1

$\Rightarrow a_0 - a_1 + a_2 - a_3 + \ldots + a_{22}$

$= (2 + 3 + 1)^{11} = 6^{11}$

… (2)

Put x = 1

$\Rightarrow a_0 + a_1 + a_2 + a_3 + \ldots + a_{22} = 0$

… (3)

adding (2) & (3)

$\Rightarrow 2[a_0 + a_2 + a_4 + \ldots\ldots + a_{22}] = 6^{11}$

$$\Rightarrow a_0 + a_2 + a_4 + \ldots + a_{22} = \frac{6^{11}}{2} = \frac{6.6^{10}}{2} = 3.6^{10}$$

Q.17 **(C)**

$$= a \sum_{r=1}^{n} (-1)^{r-1}.\ {}^{n}C_r - \sum_{r=1}^{n} r.\ {}^{n}C_r \left(-1\right)^{r-1} = a[{}^{n}C_1 - {}^{n}C_2$$

$$+ {}^{n}C_3 \ldots\ldots + (-1)^{n-1}.{}^{n}C_n] - n \sum_{r=1}^{n} (-1)^{r-1}\ {}^{n-1}C_{r-1}$$

$= a(1) - n[{}^{n-1}C_0 - {}^{n-1}C_1 + \ldots\ldots. + (-1)^{(n-1)}\ {}^{n-1}C_{n-1}]$

$= a - n(0) = a$

Q.18 **(A)**

$3\ .\ {}^{n}C_0 - 8\ .\ {}^{n}C_1 + 13\ .\ {}^{n}C_2 - 18\ {}^{n}C_3 + \ldots.$ up to $(n + 1)$ terms

$(1 + x^5)^n = C_0 + C_1x^5 + C_2x^{10} + \ldots\ldots + C_nx^{5n}$

Multiplying by x^3 and differentiating w.r.t. x

$x^3\ .\ n(1 + x^5)^{n-1}\ .\ 5x^4 + 3x^2\ (1 + x^5)^n = 3C_0\ x^2 + 8C_1\ x^7 + 13C_2\ x^{12} + \ldots\ldots + (5n + 3)\ C_n\ x^{5n+2}$

Now put $x = -1$

$3C_0 - 8C_1 + 13\ C_2 + \ldots\ldots\ldots. + (n + 1)$ terms $= 0$

Q.19 **(B)**

$$(1 + x)^n = \sum_{r=0}^{n} a_r\, x^r = a_0 + a_1x + \ldots\ldots. + a_nx^n$$

$$b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{n-r+1}{r} = \frac{n+1}{r}$$

$$\prod_{n=1}^{n} b_r = b_1b_2\ldots\ldots..b_n = \frac{(n+1)^n}{1.2.3\ldots\ldots n} = \frac{(101)^{100}}{100!}$$

$\Rightarrow \quad n = 100$

Q.20 **(D)**

$$\left(1 + 2\sqrt{x}\right)^{40} = {}^{40}C_0 + {}^{40}C_1\ 2\sqrt{x}\ + \ldots.. + {}^{40}C_{40}\ (2\sqrt{x})^{40}$$

$$(1 - 2\sqrt{x})^{40} = {}^{40}C_0 - {}^{40}C_1\ 2\sqrt{x} + \ldots\ldots + {}^{40}C_{40}\ (2\sqrt{x})^{40}$$

$$(1 + 2\sqrt{x})^{40} + (1 - 2\sqrt{x})^{40}$$

$$= 2\ [{}^{40}C_0 + {}^{40}C_2\ (2\sqrt{x})^2 + \ldots\ldots + {}^{40}C_{40}\ (2\sqrt{x})^{40}\ ]$$

Putting $x = 1$

$${}^{40}C_0 + {}^{40}C_2(2)^2 + \ldots\ldots. + {}^{40}C_{40}\ (2)^{40} = \frac{3^{40}+1}{2}$$

Q.21 **(B)**

$f(n) = 10^n + 3.4^{n+2} + 5$

put $n = 1$

$f(1) = 10 + 192 + 5 = 207$ this is divisible by 3 and 9

Q.22 **(C)**

$$\left\{\frac{3^{1001}}{82}\right\} = \left\{\frac{3.(82-1)^{250}}{82}\right\}$$

$$= \left\{\frac{3.[{}^{250}C_0(82)^{250} + {}^{250}C_1(82)^{249}(-1) + \ldots.. + {}^{250}C_{250}]}{82}\right\}$$

$$= \frac{3}{82}$$

Q.23 **(C)**

$$\left(\left(x + \frac{1}{x}\right)^2 - 1\right)^n = {}^{n}C_0 \left(x + \frac{1}{x}\right)^{2n} - {}^{n}C_1$$

$$\left(x + \frac{1}{x}\right)^{2n-2} + \ldots. + {}^{n}C_n\ (-1)^n$$

Total number of terms = 2n + 1

Q.24 (B)

$(1 + x + 2x^2)^{20} = a_0 + a_1x + \ldots\ldots\ldots + a_{40}x^{40}$

$x = 1$, then $a_0 + a_1 + \ldots\ldots\ldots + a_{40} = 4^{20}$

$x = -1$, then $a_0 - a_1 + a_2 - \ldots\ldots\ldots + a_{40} = 2^{20}$

$2^{20} + 2^{40} = 2[a_0 + a_2 + \ldots\ldots\ldots + a_{38} + a_{40}]$

$\Rightarrow\ a_0 + a_2 + \ldots\ldots\ldots + a_{38} = 2^{19} + 2^{39} - 2^{20}$

$= 2^{19}\ (2^{20}-1) \quad \because \quad a_{40} = a^{20}$

Q.25 **(B)**

$(1 + x + x^2 + \ldots\ldots + x^9)^{-1}\ (|x| < 1)$

$$= \left[\frac{1\cdot\left(1 - x^{10}\right)}{1 - x}\right]^{-1} = \frac{(1-x)}{(1-x^{10})} = (1 - x)\ (1 - x^{10})^{-1}$$

$= (1 - x)\ [1 + (x^{10}) + (x^{10})^2 + (x^{10})^3 + \ldots\ldots\ \infty]$

$= (1 - x)\ [1 + x^{10} + x^{20} + x^{30} + x^{40} + \ldots\ldots + x^{400} + x^{410} + \ldots\ldots\ \infty]$

= coeff of x^{401} is (-1)

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(A, B, C, D)**

$$\left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20}$$

$T_{r+1} = {}^{20}C_r(4^{1/3})^{20-r}(6^{-1/4})^r$

For rational terms

$20 - r = 3k$ & $r = 4p$, where $k, p \in I$

$\Rightarrow\ r = 20\ \ \&\ \ r = 8$

$\therefore$ no. of rational terms = 2

$\therefore$ no. of irrational terms = 19

Q.2 **(B,C,D)**

$\left(x^{2/3} - \frac{1}{\sqrt{x}}\right)^{30}$ term x^{13}

$$T_{r+1} = {}^{30}C_r x^{\frac{2}{3}(30-r)} (+x)^{-\frac{1}{2}r} (-1)^r = {}^{30}C_r (-1)^r \, x^{\frac{120-7r}{6}}$$

$$\frac{120-7r}{6} = 13 \Rightarrow \frac{120-78}{7} = r \Rightarrow r = 6$$

$$= {}^{30}C_6 = \frac{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25}{6\cdot 5\cdot 4\cdot 3\cdot 2}$$

which is divisibe by 29, 63, 65

Q.3 **(B,C,D)**

$$\left(x^3 + 3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11} = \left(x^3 + \frac{3}{x^3}\right)^{11}$$

$$T_{r+1} = {}^{11}C_r (x^3)^{11-r} \; 3^r \left(x^3\right)^{-r} = {}^{11}C_r \; 3^r \, (x^3)^{11-2r}$$

$$= {}^{11}C_r \, 3^r \, (x^3)^{11-2r} = {}^{11}C_r \, 3^r \, x^{33-6r}$$

(A) $33 - 6r = 2 \Rightarrow \frac{31}{6} = r \Rightarrow$ Not possible

(B) x^2 doesn't appear

(C) $33 - 6r = -3 \Rightarrow 36 = 6r \Rightarrow r = 6$

(x^{-3}) term exist/appear in exp.

(D) for x^3, r = 5, & x^{-3}, r = 6

$$\frac{T_6}{T_7} = \frac{{}^{11}C_5 \; 3^5}{{}^{11}C_6 \; 3^6} = \frac{1}{3}$$

Q.4 **(B, C, D)**

6th term in the Expension of $\left[\frac{3}{2} + \frac{x}{3}\right]^n$ for x = 3

is numerically greatest $\frac{n+1}{\left|\frac{x}{y}\right|+1} - 1 \leqslant r \leqslant \frac{n+1}{\left|\frac{x}{y}\right|+1}$

$$\Rightarrow \frac{n+1}{\frac{3}{2}+1} - 1 \leqslant 5 \leqslant \frac{n+1}{\frac{3}{2}+1}$$

$$\Rightarrow \frac{2(n+1)}{5} - 1 \le 5 \le \frac{2(n+1)}{5}$$

$$\Rightarrow \frac{2(n+1)}{5} \leqslant 6 \text{ and } \frac{2(n+1)}{5} \geqslant 5$$

$$\Rightarrow n + 1 \leqslant 15 \text{ and } n+1 \geqslant \frac{25}{2}$$

$$\Rightarrow n \leqslant 14 \text{ and } n+1 \geqslant \frac{23}{2}$$

$\Rightarrow 11.4 \leqslant n \leqslant 14$

$n \in N \Rightarrow n = 12, 13, 14$

for these value of n 6th term is greatest term

Q.5 **(A, B)**

$(1 + x^2)^2 (1 + x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$\Rightarrow (1 + x^4 + 2x^2)(1 + x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$$\Rightarrow \left(1 + x^4 + 2x^2\right)\left[1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots\right] = A_0 + A_1x + A_2x^2 + \ldots.$$

by comparism

$A_0 = 1$

$$A_1 = n \;\&\; A_2 = \frac{n(n-1)}{2!} + 2 = \frac{n^2 - n + 4}{2}$$

$2A_1 = A_0 + A_2$

$$\Rightarrow 2n = 1 + \frac{n^2 - n + 4}{2}$$

$\Rightarrow 4n = 2 + n^2 - n + 4$

$\Rightarrow n^2 - 5n + 6 = 0$

$\Rightarrow (n - 2)(n - 3) = 0$

$\Rightarrow n = 2$ or $n = 3$

Q.6 **(A, C, D)**

$\left(9 + \sqrt{80}\right)^n = I + f$

$\left(9 - \sqrt{80}\right)^n = f'$

$2[{}^nC_0 (9)^n + {}^nC_2 (9)^{n-2} (\sqrt{80})^2 + \ldots.] = I + f + f'$

$\therefore$ I = 2(integer) – 1 $\quad (\because f + f' = 1)$

$\therefore$ $(I + f)(1 - f) = 1$

Q.7 **(A,B,C)**

$101^{100} - 1 = (1 + 100)^{100} - 1$

$= {}^{100}C_0 + {}^{100}C_1(100)^1 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}(100)^{100} - 1$

$= 100 \times 100 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}\,100^{100}$

$= 10000\,[1 + {}^{100}C_2 + {}^{100}C_3(100) + \ldots + {}^{100}C_{100}\,100^{98}]$

= 10000 [Integer]

which is divisible by 100, 1000, 10000

Q.8 **(A, C)**

$7^9 + 9^7 = (8 - 1)^9 + (8 + 1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7 \ldots..+ {}^9C_8(8) - {}^9C_9 + {}^7C_0(8)^7 + \ldots\ldots + {}^7C_6(8) + {}^7C_7$

This is divisible by 64 & 16

Q.9 **(A, D)**

Constant term in $P_1(x)$ is 4

If the constant term in $P_k(x)$ is also 4, then

$P_k(x) = 4 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots$

and $P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + \ldots. + 2)^2$

Q.10 **(A, B, C)**

$$\text{General term} = \frac{10!}{r_1!\,r_2!\,r_3!}(1)^{r_1}(2x)^{r_2}(3x^2)^{r_3}$$

a_1 = Coeff. of x

$r_2 + 2r_3 = 1 \Rightarrow r_2 = 1, r_1 = 9, r_3 = 0$

$\therefore \quad a_1 = \frac{10!}{1!\,9!}(2)^1 = 20$

a_2 = Coeff. of x^2

$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2, r_1 = 8, r_3 = 0$

$r_2 = 0, r_1 = 9, r_3 = 1$

$a_2 = \frac{10!}{2!\,8!}(2)^2 + \frac{10!}{9!\,1!}(3) = 210$

a_4 = coeff. of x^4

$r_2 + 2r_3 = 4 \Rightarrow r_2 = 4, r_1 = 6, r_3 = 0$

$r_2 = 2, r_1 = 7, r_3 = 1$

$r_2 = 0, r_1 = 8, r_3 = 2$

$a_4 = \frac{10!}{4!\,6!}(2)^4 + \frac{10!}{2!\,7!\,1!}(2)^2(3) + \frac{10!}{8!\,2!}(3)^2 = 8085$

$a_{20} = 3^{10}$

Q.11 **(A, B)**

$(x + y + z)^{25}$

General term = $\frac{25!}{r_1!\,r_2!\,r_3!}\, x^{r_1} y^{r_2} z^{r_3}$

Putting $r_3 = k$, $r_2 = r - k$ and $r_1 = 25 - r$

$= \frac{25!}{(25-r)!(r-k)!(k)!} \times \frac{r!}{r!} \times x^{25-r}\, y^{r-k}\, z^k = {}^{25}C_r \,.\, {}^{r}C_k \,.\, x^{25-r}\, y^{r-k}\, z^k$

$r_1 + r_2 + r_3 = 25$

$\therefore$ coefficient of $x^8\, y^9\, z^9$ is 0

Comprehension # 1 (Q. No. 12 to 14)

Q.12 **(B)**

Required sum = coefficient of x^n in

$(1 + x)^n\,(1 + x)^n$

= coefficient of x^n in $(1 + x)^{2n}$

$= {}^{2n}C_n$

Q.13 **(D)**

$\because\ {}^{30}C_r.\, {}^{20}X_0 + {}^{30}C_{r-1}.\, {}^{20}C_1 . + \ldots.. + {}^{30}C_0.{}^{20}C_r$

= coefficient of $x^r\,(1 + x)^{30}\,(1 + x)^{20}$

= coefficient of x^r in $(1 + x)^{50}$

$= {}^{50}C_r$

$\because\ {}^{50}C_r$ is maximum

$\therefore \quad r = \frac{50}{2} = 25$

Q.14 **(A)**

$\because\ {}^{20}C_r.\, {}^{10}C_0 + {}^{20}C_{r-1}.\, {}^{10}C_1 + \ldots + {}^{20}C_0 \,.\, {}^{10}C_r$

= coefficient of x^r in$(1 +x)^{20}\,(1 + x)^{10}$

= coefficient of $x^r\,(1 + x)^{30}$

$= {}^{30}C_r$

$\because\ {}^{30}C_r$ is minimum

$\therefore\ r = 0$

Comprehension # 2 (Q. No. 15 to 17)

Q.15 **(C)**

$(1 + i)^n = C_0 + C_1 i + C_2 i^2 + C_3 i^3 + \ldots.$

$\Rightarrow \quad (1 + i)^n = (C_0 - C_2 + C_4 - C_6 + \ldots\ldots) + i(C_1 - C_3 + C_5 - \ldots.) \qquad \ldots.(i)$

$\Rightarrow \quad C_0 - C_2 + C_4 - C_6 + \ldots.$ = Real part of $(1 + i)^n$

= Real part of $\left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^n$

= Real part of $2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) = 2^{n/2}\cos\frac{n\pi}{4}$.

Q.16 **(B)**

From the previous question on taking modulus of equation $\qquad \ldots..(i)$

$(C_0 - C_2 + C_4 - \ldots\ldots)^2 + (C_1 - C_3 + C_5 - \ldots\ldots)^2 = ((\sqrt{2})^n)^2 = 2^n$

Q.17 **(C)**

Add the expansion of $(1 + 1)^n$, $(1 + \omega)^n$ and $(1 + \omega^2)^n$

$(1 + \omega)^n = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^n = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^n$

$(1 + \omega^2)^n = \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^n = \left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^n$

Comprehension # 3 (Q. No. 18 to 20)

Q.18 **(D)**

The expression $(2 + x)^2\,(3 + x)^3\,(4 + x)^4 = (x + 2)(x + 2)\,(x + 3)(x + 3)(x + 3)(x + 4)(x + 4)(x + 4)(x + 4)$

$= x^9 + (2 + 2 + 3 + 3 + 3 + 4 + 4 + 4 + 4)\,x^8 + \ldots\ldots\ldots\ldots.$

$\Rightarrow$ Co-efficient of $x^8 = 29$

Q.19 **(C)**

Expression = $x.\, x^2.\, x^3 \ldots\ldots\ldots..x^{20}$

$\left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x^2}\right)\left(1 - \frac{3}{x^3}\right)\ldots\ldots.\left(1 - \frac{20}{x^{20}}\right)$

Let $E = \left(1 - \frac{1}{x}\right)\left(1 - \frac{2}{x^2}\right)\left(1 - \frac{3}{x^3}\right)\ldots\ldots.\left(1 - \frac{20}{x^{20}}\right)$

Now Co-efficient of x^{203} in original expression

$\Rightarrow$ Co-efficient of x^{-7} in E.

But

$E = 1 - \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \ldots\ldots\right) +$

$$\left(\frac{1}{x}.\frac{6}{x^6}+\frac{2}{x^2}.\frac{5}{x^5}+\frac{3}{x^3}.\frac{4}{x^4}+......\right)-\left(\frac{1}{x}.\frac{2}{x^2}.\frac{4}{x^4}+......\right)$$

= Co-efficient of $x^{-7} = -7 + 6 + 10 + 12 - 8 = 13$

Q.20 **(C)**

The Co-efficient of $x^{98} = (1.2 + 2.3 + ...99.100)$

= Sum of product of first 100 natural numbers taken two at a time

$$= \frac{1}{2}[(1 + 2 + 3 + ...+ 100^2)^2 - (1^2 + 2^2 + 3^2 + ...+ 100^2)]$$

Q.21 (A) → (q, s), (B) → (p, q, r), (C) → (s), (D) → (p, s)

(A) We have, T_{r+1} =

$$\frac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right).....\left(\frac{7}{2}-r+1\right)x^r}{r!}$$

This will be the first negative term when

$\frac{7}{2} - r + 1 < 0$ i.e. $r > \frac{9}{2}$

Hence r = 5.

(B) We have, $T_{r+1} = {}^5C_r\,(y^2)^{5-r}\left(\frac{1}{y}\right)^r = {}^5C_r\,y^{10-3r}$

Now, $10 - 3r = 1 \Rightarrow r = 3$

So, coefficient of $y = {}^5C_3 = 10$

(C) Obviously a prime number.

(D) We have : $(1 + 2x + 3x^2 + 4x^3 + ...)^{1/2}$

$= [(1 - x)^{-2}]^{1/2} = (1 - x)^{-1} = 1 + x + x^2 + ... + x^n + ...$

Hence, coefficient of x^4 = 1 c = 1, hence c + 1 = 2

Q.22 (A) → (p); (B) → (q,t); (C) → (r,s)

(A) $\because (1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)^{20}$

$= \{(1 = x)^5\}^{20} = (1 + x)^{100}$

$\therefore \lambda = 100 + 1 = 101$

Then $3^\lambda = 3^{101} = 3.3^{100} = 3(9)^{50} = 3(10 - 1)^{50}$

$= 3\{(10)^{50} - {}^{50}C_1\,(10)^{49} +$

$-{}^{50}C_2\,(10)^{48} - ...- {}^{50}C_{49}\,(10) + 1\}$

$= 3\{100\mu + 1\}$ $300\mu +$ (μ is + ve integer)

$\therefore$ Last two digits 03

$\therefore$ O = 3, T = 0 ⇒ O + T = 3(p)

(B) $\left(x^2 + 1 + \frac{1}{x^2}\right)^{100}$

$\therefore \lambda = 2 \times 100 + 1 = 201$

Then $7^\lambda = 7^{201} = 7.7^{200} = 7.(7^2)^{100} = 7.(49)^{100}$

$= 7(50 - 1)^{100}$

$= 7\{(50)^{100} - {}^{100}C_1(50)^{99} + 100C_2(50)^{98} - ... - {}^{100}C_{99}(50) + 1\}$

$= 7\{100\mu + 1\} = 700\mu + 7$

($\because$ μ is + ve integer)

$\therefore$ Last two digits 07

$\therefore$ O = 7, T = 0

⇒ O + T = 7 and O – T = 7 (q, t)

(C) $\because (1 + x)^{101}\,(1 + x)(1 + x^2 - x)^{100}$

$= (1 + x)\{(1+ x)(1 + x^2 -x)\}^{100}$

$= (1 + x)(1 + x^3)^{100}$

$= (1 + x)\{1 + {}^{100}C_1\,x^3 + {}^{100}C_2x^6 + {}^{100}C_3x^9 + ... + {}^{100}C_{100}x^{303}\}$

$= 1 + {}^{100}C_1x^3 + {}^{100}C_2x^6 + {}^{100}C_3x^9 + ... + {}^{100}C_{100}x^{303}$
$+ x + {}^{100}C_2x^7 + {}^{100}c_3x^{10} + ... + {}^{100}c_{100}x^{304}$

$\therefore \lambda = 1 + 100 + 101 = 202$

$\Rightarrow 9^l = 9^{202} = (10 -1)^{202}$

$= (10)^{202} - {}^{202}C_1(10)^{201} + {}^{202}C_2(10)^{200} - ... - {}^{202}C_{201}(10) + {}^{202}C_{202}$

$= 100\mu - 2020 + 1$ (μ is + ve integer)

$= 100(\mu - 21)$ 81

$= 100v + 81$ (v is + ve integer)

$\therefore$ Last two digits 81

$\therefore$ O = 1 and T = 8

⇒ O + T = 9 and T – O = 7 (r, s)

Q.23 (A) → (q), (B) → (s), (C) → (p), (D) → (r)

(A) $I + f = (7 + 4\sqrt{3})^{2n}$

Here $(7 - 4\sqrt{3})^{2n} = f' = 1 - f$

$(I + f)(1 - f) = 1$

(B) $T_2 = {}^nC_1\,(x)^{n-1}\,.\,a = 240$ ....(i)
$T_3 = {}^nC_2\,(x)^{n-2}\,a^2 = 720$ ....(ii)
$T_4 = {}^nC_3\,(x)^{n-3}\,a^3 = 1080$ ....(iii)
From (i) and (ii)

Here $\frac{{}^nC_1(x)^{n-1}a}{{}^nC_2x^{n-2}a^2} = \frac{2x}{(n-1)a} = \frac{240}{720} = \frac{1}{3}$

$\Rightarrow 6x = (n - 1)a$

From (ii) and (iii)

$9x = 2(n - 2)\,a$

On dividing $\frac{3}{2} = \frac{2(n-2)}{(n-1)} \Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$

(C) $C_0C_4 - C_1C_3 + C_2\,C_2 - C_3C_1 + C_4C_0 = 2 - 2.4.4 + 6.6$
$= 6$

(D) $\sqrt{\frac{x}{x+y}}\,\sqrt{\frac{x}{x-y}} = \left(\frac{1}{1+\frac{y}{x}}\right)^{1/2}\left(\frac{1}{1-\frac{y}{x}}\right)^{1/2}$

$= \left(1-\frac{y^2}{x^2}\right)^{-1/2} = 1 + \frac{1}{2}\,.\,\frac{y^2}{x^2}$

⇒ k = 2

NUMERICAL VALUE BASED

Q.1 (10)

$T_6 = {}^8C_5 \left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 = 5600 \quad \Rightarrow \frac{1}{x^8}$

$x^{10} (\log_{10} x)^5 = 100 \quad \Rightarrow x = 10$

Q.2 (2)

$$T_4 = {}^8C_3 \left(5^{\frac{1}{5}\log_5(4^x+44)}\right)^5 \left(\frac{1}{5^{\frac{1}{3}\log_5(2^{x-1}+7)}}\right)^3$$

$\Rightarrow \quad {}^8C_3(4^x + 44) \left(\frac{1}{2^{x-1}+7}\right) = 336$

$\Rightarrow \quad \frac{4^x + 44}{2^{x-1}+7} = 6$

$\Rightarrow \quad 4^x + 44 = 3.2^x + 42$

$\Rightarrow \quad (2^x)^2 - 3.2^x + 2 = 0$

$\Rightarrow \quad (2^x - 1)(2^x - 2) = 0 \quad \Rightarrow \quad x = 0$ & 1

Q.3 (32)

$P = {}^{2n}C_n$ and $Q = {}^{2n-1}C_n \quad \Rightarrow \quad \frac{P}{Q} = 2$

; $\left(1+\frac{P}{Q}\right)^5 = (1 + 2)^5 = 3^5$

Q.4 (50)

Co-efficient of x^{50}

$S = (1 + x)^{1000} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} + ... + 1001x^{1000}$ ....(i)

$\frac{xS}{1+x} = x(1 + x)^{999} + 2x^2(1 + x)^{998}.... + 1000x^{1000} +$

$\frac{1001x^{1001}}{(1+x)}$ ....(ii)

____

$\frac{S}{1+x} = (1 + x)^{1000} + x(1 + x)^{999} + ... + x^{1000} -$

$\frac{1001\,x^{1001}}{1+x}$

$$\Rightarrow \quad \frac{S}{1+x} = (1 + x)^{1000} \left[\frac{1-\left(\frac{x}{1+x}\right)^{1001}}{1-\frac{x}{1+x}}\right] -$$

$\frac{1001\,x^{1001}}{(1+x)}$

$\Rightarrow \quad S = (1 + x)^{1002} - x^{1001}(1 + x) - 1001\,x^{1001}$

Co-efficient of $x^{50} = {}^{1002}c_{50}$

Q.5 (n = 12)

For T_9 to be the numerically greatest term, r =

$$\left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] = \left[\frac{n+1}{1+\left|\frac{1}{2}\right|}\right] = 8$$

$\Rightarrow \quad 8 < \frac{2(n+1)}{3} < 9 \Rightarrow 11 < n < 12.5 \Rightarrow n =$ 12

Q.6 (2)

${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$r^2 = 3r$ or $\quad r = 0, 3$

Q.7 (12)

$$\sum_{k=1}^{n} k^3 \left(\frac{n-k+1}{k}\right)^2 = \sum_{k=1}^{n} k(n-k+1)^2 =$$

$$\sum_{k=1}^{n} (n^2k + k^3 + k - 2nk^2 + 2nk - 2k^2)$$

$$= \frac{(n+1)^2.n(n+1)}{2} + \left[\frac{n(n+1)}{2}\right]^2 -$$

$$\frac{2(n+1)n(n+1)(2n+1)}{6} = \frac{n(n+1)^2(n+2)}{12}$$

Q.8 (3)

$$\sum_{m=p}^{n} {}^nC_m.{}^mC_p = \sum_{m=p}^{n} \frac{n!}{m!(n-m)!} \times \frac{m!}{p!(m-p)!} =$$

$$\sum_{m=p}^{n} {}^nC_p.{}^{n-p}C_{m-p}$$

$= {}^nC_p\,[{}^{n-p}C_0 + {}^{n-p}C_1 + ...+ {}^{n-p}C_{n-p}] = {}^nC_p\,2^{n-p}$;
where n = 100 and p = 97.

Q.9 **(9)**

$\because \quad (1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1x + ...+ {}^{2n}C_{2n}x^{2n}$

differentiating it

$2n(1+x)^{2n-1} = {}^{2n}C_1 + 2.\,{}^{2n}C_2x + …………+ 2n\,{}^{2n}C_{2n}x^{2n-1}$

Again $(x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + ...+ {}^{2n}C_{2n}$

Required expression = coefficient of x^{2n-1} in $2n\,(1+x)^{4n-1}$

$= 2n.\,{}^{4n-1}C_{2n-1}$

Q.10 (5)

$$\sum_{r=0}^{n}\frac{(-1)^r.C_r}{(r+1)(r+2)(r+3)} =$$

$$\frac{-1}{(n+1)(n+2)(n+3)}\left\{\sum_{r=0}^{n} {}^{n+3}C_{r+3}(-1)^{r+3}\right\}$$

$$\frac{-1}{(n+1)(n+2)(n+3)}$$

$$\left[\begin{array}{l}(1-1)^{n+3} - \\ \left\{{}^{n+3}C_0(-1)^0 + {}^{n+3}C_1(-1)^1 + {}^{n+3}C_2(-1)^2\right\}\end{array}\right]$$

$$\frac{1}{(n+1)(n+2)(n+3)}(n+2)\times\frac{(n+1)}{2} = \frac{1}{2(n+3)}$$

KVPY
PREVIOUS YEAR'S

Q.1 (Bouns)

Coeff. Of x^{2012}

$$\frac{(1+x)^2}{(1+x^2)(1-x^2)}$$

$= (1 + x)^2 (1 - x^4)^{-1}$

$= (1 + 2x + x^2) (1 - x^4)^{-1}$

Coeff. Of x^{2012} + 2Coeff of x^{2011} + Coeff of x^{2010} in the expansion of $(1- x^4)^{-1}$ x^{2011} and x^{2010} not possible in $(1 - x^4)^{-1}$

= only coeff. Of x^{2012} in the expassion of

$(1 - x^4)^{-1}$ ${}^{1+503-1}C_{503} = 1$

Q.2 (C)

$$(n + 1)^{1/3} - n^{1/3} < \frac{1}{12}$$

$$(n + 1) - n - 3 (n + 1)^{1/3} n^{1/3} ((n + 1)^{1/3} - n^{1/3}) < \left(\frac{1}{12}\right)^3$$

$$1 - 3n^{1/3} (n + 1)^{1/3} \times \frac{1}{12} < \frac{1}{(12)^3}$$

$(12)^3 - 3.(12)^2 n^{1/3} (n + 1)^{1/3}$

$(12)^3 - 1 < 3. (12)^2 n^{1/3} (n + 1)^{1/3}$

$$\frac{1727}{3\times144} < n^{1/3}(n+1)^{1/3}$$

$$n(n + 1) > \left(\frac{1727}{3\times144}\right)^3$$

$n (n + 1) > 63.88$

$n = 8$

Q.3 (D)

$P(x) = 1 + x^2 + x^4 + x^6 + \ldots\ldots + x^{22} = (1 + x^2) (1 + x^4) (1 + x^4 + x^8) (1 - x^4 + x^8)$

$Q(x) = 1 + x + x^2 + x^3 + \ldots\ldots + x^{11} = (1 + x) (1 + x^2) (1 + x^4 + x^8)$

$$\frac{P(x)}{Q(x)} = \frac{(1+x^4)(1-x^4+x^8)}{(1+x)} = \frac{1-x^4+x^8+x^4-x^8+x^{12}}{1+x} = \frac{1+x^{12}}{1+x}$$

Remainder. When $(1 + x^{12})$ is divided by $(1 + x)$ is = 2

Now remainder P(x) divided by Q(x)

$= 2(1 + x^2) (1 + x^4 + x^8)$

$= 2(1 + x^2 + \ldots\ldots. + x^{10})$

Q.4 (C)

$I + F_2 = (\sqrt{2017} + 44)^{2017}$

$F_2' = (\sqrt{2017} - 44)^{2017}$; $0 < F_2' < 1$

$I + F_2 - F_2' = 2\left[{}^{2017}C_1(\sqrt{2017})^{2016}(44) + \ldots\right]$

$F_2 = F_2'$

$F_2 = (0.911)^{2017}$

Now, $F_1 = (44 - \sqrt{2017})^{2017} = -(0.911)^{2017}$

Fractional part can not –ve.

So, $F_1 = 1 - (0.911)^{2017}$

So, $F_1 + F_2 = 1$

1 lie between 0.9 and 1.35

Q.5 (A)

$y = P(x) = a_0 + a_1 \times a_2 x^2 + \ldots.. + a_n x^n$ $a_0, a_1, a_2, a_3, \ldots\ldots. a_n \in 1$

$2 = P (2)$ ......(1)

$5 = P (4)$ ......(2)

By (1) & (2)

$$\Rightarrow 3 = a_1(4-2) + a_2(4^2 - 2^2) + a_3(4^2 - 2^n)$$

Clearly RHS is even and LHS is odd no polynomial exists.

Q.6 **(A)**

$$a = b = \sum_{n=101}^{200} 2^n \sum_{k=101}^{n} \frac{1}{k!}$$

$$= \frac{2^{101}}{101!} + 2^{102}\left(\frac{1}{101!} + \frac{1}{102!}\right) + 2^{103}\left(\frac{1}{101!} + \frac{1}{102!} + \frac{1}{103!}\right) + \ldots$$

$$+ 2^{200}\left(\frac{1}{101!} + \frac{1}{102!} + \ldots + \frac{1}{200!}\right)$$

$$\frac{2^{101} + \ldots + 2^{200}}{101!} + \frac{2^{102} + \ldots + 2^{200}}{102!} + \ldots + \frac{2^{200}}{200!}$$

$$= \frac{2^{101}(2^{100} - 1)}{101!} + \frac{2^{102}(2^{99} - 1)}{102!} + \ldots + \frac{2^{200}}{200!}$$

$$= \left(\frac{2^{201}}{101!} - \frac{2^{101}}{101!}\right) + \left(\frac{2^{201}}{102!} - \frac{2^{102}}{102!}\right) + \ldots + \left(\frac{2^{201}}{200!} - \frac{2^{200}}{200!}\right)$$

$$= \sum_{n=101}^{200} \frac{2^{201} - n^n}{n!} = b$$

$$\therefore \frac{a}{b} = 1$$

Q.7 **(A)**

$(2021)^{2020} = (1 + 2020)^{2020}$

${}^{2020}C_0 + {}^{2020}C_1.2020 + {}^{2020}C_2 . 2020^2 + ... + {}^{2020}C_{2020}.2020^{2020}$

$1 + (2020)^2 + {}^{2020}C_2.2020^2 + ... + {}^{2020}C_{2020}.2020^{2020}$

$1 + (2020)^2 (1 + {}^{2020}C_2 + ... + (2020)^{2018})$

$1 + (2020)^2.\lambda$

Hence $(2021)^{2020} = \lambda(2020)^2 + 1$

Hence remainder = 1

Q.8 **(D)**

If n is a composite number (Take n = 4)

For n = 4, n does not divide (n . 1)!

Hence Ist statement is false

(II) $n^3 + 2n^2 + n = n(n + 1)^2$

$n(n + 1)^2$ divides n!

If n is such that (n + 1) is a prime number (Take n = 6) so $n(n + 1)^2$ does not divide n! but there are infinite values of n (n = 104, 109, 114, ...) for which $n(n + 1)^2$ divide n! but it is not true for every natural numbers.

JEE MAIN PREVIOUS YEARS

Q.1 (1)

$x = 4k + 3 \; ; k \in W$

$\therefore\ (2020 + 4k + 3)^{2022} = (8\lambda + 1)^{1011}$

$\therefore\ (8\lambda + 1)^{1011} = {}^{1011}C_0 + \underbrace{{}^{1011}C_1(8y) + \text{--------}}_{\text{multiple of 8.}}$

$\therefore$ Remainder on dividing by 8 is 1

Q.2 (2)

$S = {}^2C_2 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\therefore\ {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$$= \frac{(n + 1)!}{3!(n - 1)} + \frac{(n + 1)!}{3!(n - 1)}$$

$$= \frac{(n + 2)(n + 1)n}{6} + \frac{(n + 1)(n)(n - 1)}{6}$$

$$= \frac{(n + 1)}{6}\ (2n + 1)$$

Q.3 (1)

$$T_{r+1} = {}^{10}C_r\ (t\ x^{1/5})^{10-r}\left(\frac{(1-x)^{1/10}}{t}\right)^r$$

$10 - r - r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5\ x(1 - x)^{1/2}$

$$\frac{d(T_6)}{dx} = {}^{10}C_5\left((1-x)^{1/2} + \frac{-x}{2\sqrt{1-x}}\right) = 0$$

$$2(1 - x) - x = 0 \Rightarrow x = \frac{2}{3}$$

$$\text{Maximum } T_6 = {}^{10}C_3\ \frac{2}{3}\left(\frac{1}{3}\right)^{1/2} = 56\sqrt{3}$$

Q.4 (59)

General term $= (30 - r).{}^{30}C_r$

$$\text{L.H.S} = \sum_{r=0}^{30}(30-r)\ .{}^{30}C_r$$

$$= 30\sum_{r=0}^{30} 30\ Cr - \sum_{r=0}^{30} r^{30}.C_r$$

$= 30.2^{30} - 30.2^{29}$

$= 30.2^{29}$

So n = 30, m = 29

m + n = 59

Q.5 (2)

$S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - \ldots\ldots\ldots - 15.{}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r\ .r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$$

$= 15\ (-{}^{14}C_0 + {}^{14}C_1 - \ldots\ldots - {}^{14}C_{14}) = 15\ (0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + \ldots\ldots\ldots + {}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + \ldots\ldots\ldots + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$S_1 + S_2 = 2^{13} - 14$

Q.6 (3) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$

put 2n+1=r, where r=3, 5, 7, ...

$$\Rightarrow n = \frac{r+1}{2}$$

$$\frac{n^2-6n+10}{(2n+1)!} = \frac{\left(\frac{r-1}{2}\right)^2 + 3r - 3 + 10}{r!} = \frac{r^2+10r+29}{4r!}$$

$$\text{Now } \sum_{r=3,5,7}\frac{r(r-1)+11r+29}{4r!} = \frac{1}{4}$$

$$\sum_{r=3,5,7}\left(\frac{1}{(r-2)!} + \frac{11}{(r-1)!} + \frac{29}{r!}\right)$$

$$= \frac{1}{4}\left\{\left(\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots\right) + 11\left(\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots..\right) + 29\left(\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\ldots.\right)\right\}$$

$$= \frac{1}{4}\left\{\frac{e-\frac{1}{e}}{2} + 11\left(\frac{e+\frac{1}{e}-2}{2}\right) + 29\left(\frac{e-\frac{1}{e}-2}{2}\right)\right\}$$

$$= \frac{1}{8}\left\{e - \frac{1}{e} + 11e + \frac{11}{e} - 22 + 29e - \frac{29}{e} - 58\right\}$$

$$= \frac{1}{8}\left\{41e - \frac{19}{e} - 80\right\}$$

Q.7 **(1)**

$(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k\,; \qquad 0 \le r \le 60$

$\quad 0 \le 8k \le 60$

$0 \le k \le \frac{60}{8}$

$0 \le k \le 7.5$

k = 0, 1, 2, 3, 4, 5, 6, 7

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

n – 1 = 53 – 1 = 52

52 is divisible by 26.

Q.8 **(3)**

$$(1-x+x^3)^n = \sum_{j=0}^{3n} a_j\, x^j$$

$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 .... + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of a0 + a_2 + a_4.......

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$= Sum of a_1 + a_3 + a_5.......

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 .... + a_{3n}$

.......(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3$ .... $+ (-1)^{3n}a_{3n}$.......(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 .... = 1$

$a_1 + a_3 + a_5 .... = 0$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

Q.9 **(6)**

$$A = \sum_{k=0}^{n} {}^nC_k\left[\left(+++\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k \quad \left(\frac{-7}{8}\right)^k \quad \left(\frac{-15}{16}\right)^k \quad \left(\frac{-37}{32}\right)^k\right]$$

$$A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$$

$$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$$

$$A = \frac{1}{2^n}\left(\frac{1-\left(\frac{1}{2^n}\right)^5}{1-\frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1-\frac{1}{2^{5n}}\right)}{(2^n-1)}$$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly 5n = 30

n = 6

Q.10 **(1)**

${}^7C_3x^4 \quad x^{(3\log_2^x)} = 4480$

$\Rightarrow x^{(4+3\log_2^x)} \quad 2^7$

$\Rightarrow (4 \quad 3t)t \quad 7;t \quad \log_2^x$

$\Rightarrow \quad t = 1, \frac{-7}{3} \Rightarrow x = 2$

Q.11 **(4)**

$(2023 - 2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as 2023 = 17 × 17 × 9)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4\,(17k_3 + 1)$

= 17k + 4 ⇒ remainder = 4

Q.12 **(4)**

$$\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$$

$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + ... + {}^6C_6 \cdot {}^6C_0$

Now,

$(1+x)^6(1+x)^6$

$= \left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + ... + {}^6C_6x^6\right)$

$\left({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + ... + {}^6C_6x^6\right)$

Comparing coefficeint of x^6 both sides

${}^6C_0 \cdot {}^6C_6 + {}^6C_1 + {}^6C_5 + ... + {}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$

= 924

Q.13 **(4)**

$$T_{r+1} = {}^nC_r(x)^{n-r}\left(\frac{a}{x^2}\right)^r$$

$= {}^nC_r\, a^r x^{n-3r}$

${}^nC_2\, a^2 : {}^nC_3\, a^3 : {}^nC_4\, a^4 = 12 : 8 : 3$

After solving

n = 6, a = $\frac{1}{2}$

For term independent of 'x' ⇒ n = 3r

r = 2

$\therefore$ Coefficient is ${}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

Q.14 **(2)**

$(1 + x + 2x^2)^{20} = a_0 + a_1x + ...+ a_{40}x^{40}$

put x = 1, − 1

$\Rightarrow a_0 + a_1 + a_2 +.... + a^{40} = 220$

$a_0 - a_1 + a_2 +.... + a^{40} = 220$

$$\Rightarrow a_1 + a_3 +.... + a^{39} = \frac{4^{20} - 2^{20}}{2}$$

$\Rightarrow a_1 + a_3 +... + a^{37} = 239 - 219.\ a39$

$$\text{here } a^{39} = \frac{20!\left(2\right)^{19} \times 1}{19!} = 20 \times 219$$

$\Rightarrow a_1 + a_3 +... + a^{37} = 219(220 - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

Q.15 **(160)**

$$\sum_{r=1}^{10} r!\left\{(r+1)(r+2)(r+3) - 9(r+1) + 8\right\}$$

$$= \sum_{r=1}^{10}\left[\left\{(r+3)! - (r+1)!\right\} - 8\left\{(r+1)! - r!\right\}\right]$$

$= (13! + 12! – 2! – 3!) – 8(11! – 1)$

$= (12.13 + 12 – 8).11! – 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

Q.16 **(210)**

$$\left(\left(x^{1/3} + 1\right) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}}\right)\right)^{10}$$

$(x^{1/3} – x^{–1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10–r}\ (–x^{–1/2})^r$

$$\frac{10 - r}{3} - \frac{r}{2} = 0 \Rightarrow 20 – 2r – 3r = 0$$

$\Rightarrow r = 4$

$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Q.17 **(19)**

Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$$\sum_{k=0}^{10}\left(2^2 + 3k\right){}^{10}C_k = \alpha.3^{10} + \beta.2^{10}$$

$$\text{LHS} = 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10}k.\frac{10}{k}.{}^9C_{k-1}$$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \beta = 19 \Rightarrow \alpha + \beta = 19$

Q.18 **(8)**

Q.19 **(96)**

Q.20 (1)

Q.21 (210)

Q.22 (4)

Q.23 (4)

Q.24 (1)

Q.25 (98)

Q.26 (55)

Q.27 (3)

Q.28 (4)

Q.29 (2)

Q.30 (21)

Q.31 (4)

Q.32 (924)

Q.33 **(315)**

Q.34 (55)

Q.35 **(49)**

Q.36 (4)

Q.37 (1)

Q.38 **(2)**

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 **[6]**

${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5 : 10 : 14$

$$\Rightarrow \frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = \frac{10}{5} \qquad \& \quad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{14}{10}$$

$$\Rightarrow \frac{(n+5) - r + 1}{r} = 2 \qquad \&$$

$$\frac{(n+5) - (r+1) + 1}{r+1} = \frac{7}{5}$$

$$\Rightarrow \frac{n+6}{r} = 3 \qquad \& \quad \frac{n+6}{r+1} = \frac{12}{5}$$

$$\Rightarrow 3r = \frac{12}{5}(r + 1) \qquad \Rightarrow r = 4$$

$\therefore$ n + 6 = 12

$\Rightarrow n = 6$

Q.2 **(C)**

Coefficent of $x^{11} \equiv$

$$\frac{(1+x^2)^4(1+x^3)^7(1+x^4)^{12}(1-x^2)^4}{(1-x^2)^4}$$

Coefficent of $x^{11} \equiv (1 – x^8)^4\ (1 + x^4)^8\ (1 + x^3)^7\ (1 – x^2)^{–4}$

$= (1 – 4x^8)\ (1 + x^4)^8\ (7x^3 + 35x^9)\ (1 – x^2)^{–4}$

$= (7x^3 + 35x^9 – 28x^{11})\ (1 + x^4)^8\ (1 – x^2)^{–4}$

Coefficent of $x^8 = (7x + 35x^6 – 28x^8)\ (1 + 8x^4 + 28x^8)\ (1 – x^2)^{–4}$

$= (7 + 35x^6 – 28x^8 + 56x^4 + 196x^8)\ (1 – x^2)^{–4}$

Coefficent of $t^4 \equiv (7 + 56t^2 + 35t^3 + 168t^4)\ (1 – t)^{–4}$

$= 7 \cdot {}^7C_3 + 56 \cdot {}^5C_3 + 35 \cdot {}^4C_3 + 168$

$= 245 + 700 + 168 = 1113.$

Alterantive :

2x + 3y + 4z = 11

(x, y, z) = (0, 1, 2) ${}^4C_0 \times {}^7C_1 \times {}^{12}C_2$

(1, 3, 0) ${}^4C_1 \times {}^7C_3$

(2, 1, 1) ${}^4C_2 \times {}^7C_1 \times {}^{12}C_1$

(4, 1, 0) ${}^7C_1$

coefficient of x^{11} = 66 × 7 + 35 × 4 + 42 × 12 + 7

= 1113.

Q.3 **[8]**

9 = (0, 9) (1, 8), (2, 7), (3, 6), (4, 5) # 5 cases

9 = (1,2,6), (1,3,5), (2, 3, 4) # 3 cases

total = 8

Q.4 **[5]**

coefficient of x^2 in $(1+x)^2 + (1+x)^3+...+ (1+x)^{49} + (1+mx)^{50} = (3n + 1)\ {}^{51}C_3$

⇒ coefficient of x^2 in

$$\frac{(1+x)^2\left((1+x)^{48}-1\right)}{1+x-1}+(1+mx)^{50} = (3n + 1)\ {}^{51}C_3$$

⇒ ${}^{50}C_3 + {}^{50}C_2 \times m^2 = = (3n + 1)\ {}^{51}C_3$

⇒ $16 + m^2 = (3n + 1)\ 17$

⇒ $n = 5$

Q.5 **[646]**

$$X = \sum_{r=0}^{n} r.\left({}^{n}C_r\right)^2 ; n = 10$$

$$X = n.\sum_{r=0}^{n} {}^{n}C_r.{}^{n-1}C_{r-1}$$

$$X = n.\sum_{r=1}^{n} {}^{n}C_{n-r}\ .{}^{n-1}C_{r-1}$$

$$X = n.{}^{2n-1}C_{n-1}; n = 10$$

$$X = 10.{}^{19}C_9$$

$$\frac{X}{1430} = \frac{1}{143}.{}^{19}C_9$$

= 646

Q.6 [6.20]

Suppose

$$\begin{vmatrix} \dfrac{n(n+1)}{2} & n(n-1).2^{n-2}+n.2^{n-1} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\frac{n(n+1)}{2}\ .\ 4^n - n^2 (n-1).2^{2n-3} - n^2\ 2^{2n-2} = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$$

$n^2 - 3n - 4 = 0$

$n = 4$

Now $\displaystyle\sum_{k=0}^{4}\frac{{}^4C_k}{k+1} = \sum_{k=0}^{4}\frac{k+1}{5}\ .\ {}^5C_{k=1}\ \frac{1}{k+1}$

$$= \frac{1}{5}\ .\ [{}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5] = \frac{1}{5}[2^5 - 1] = \frac{31}{5}$$

= 6.20

Q.7 (A,B,D)

Solving

$$f(m, n, p) = \sum_{i=0}^{p} {}^{m}C_i\ {}^{n+i}C_p\ .{}^{p+n}C_{p-i}$$

$${}^{m}C_i.{}^{n+i}C_p.{}^{p+n}C_{p-i}$$

$${}^{m}C_i.\frac{(n+i)!}{p!(n-p+i)!}\times\frac{(n+p)!}{(p-i)!(n+i)!}$$

$${}^{m}C_i\times\frac{(n+p)!}{p!}\times\frac{1}{(n-p+i)!(p-i)!}$$

$${}^{m}C_i\times\frac{(n+p)!}{p!n!}\times\frac{n!}{(n-p+i)!(p-i)!}$$

${}^{m}C_i\ .\ {}^{n+p}C_p.{}^{n}C_{p-i}$ $\quad \{{}^{m}C_i.{}^{n}C_{p-i} = {}^{m+n}C_p\}$

$f(m,n,p) = {}^{n+p}C_p.{}^{m+n}C_p$

$$\frac{f(m,n,p)}{{}^{n+p}C_p} = {}^{m+n}C_p$$

Now

$$g(m, n) = \sum_{p=0}^{m+n}\frac{f(m,n,p)}{{}^{n+p}C_p}$$

$$g(m, n) = \sum_{p=0}^{m+n} {}^{m+n}C_p$$

$g(m, n) = 2^{m+n}$

(A) g(m,n) = q(n, m)

(B) $g(m,n+1) = 2^{m+n+1}$

$g(m+n,n) = 2^{m+1+n}$

(D) $g(2m, 2n) = 2^{2m+2n}$

$= (2^{m+n})^2$

$= (g(m,n))^2$

Complex Number

EXERCISES

ELEMENTARY

Q.1 (2)

We know that $i^2=-1$ $(i^2)^2=(-1)^2=1 \Rightarrow$ $i^{4n}=1^n$

and therefore $i^{4n-1}=-i$, $i^{4n+1}=i$

Q.2 (2)

$$\frac{i^{584}(i^8+i^6+i^4+i^2+1)}{i^{574}(i^8+i^6+i^4+i^2+1)}-1=\frac{i^{584}}{i^{574}}-1$$

$$=i^{10}-1=-1-1=-2$$

Q.3 (3)

Given that $a^2+b^2=1$, therefore

$$\frac{1+b+ia}{1+b-ia}=\frac{(1+b+ia)(1+b+ia)}{(1+b-ia)(1+b+ia)}$$

$$=\frac{(1+b)^2-a^2+2ia(1+b)}{1+b^2+2b+a^2}$$

$$=\frac{(1-a^2)+2b+b^2+2ia(1+b)}{2(1+b)}$$

$$=\frac{2b^2+2b+2ia(1+b)}{2(1+b)}=b+ia$$

Trick : Put $a=0, b=1$, $\frac{1+b+ia}{1+b-ia}=\frac{1+1+0}{1+1-0}=1$

But options (1) and (3) give 1.

So again put $a=1, b=0, \frac{1+b+ia}{1+b-ia}=\frac{1+i}{1-i}=i$

Which gives (3) only.

Q.4 (2)

$$\frac{1}{1-\cos\theta+i\sin\theta}$$

$$=\frac{1}{(1-\cos\theta)+i\sin\theta}\times\frac{(1-\cos\theta)-i\sin\theta}{(1-\cos\theta)-i\sin\theta}$$

$$=\frac{(1-\cos\theta)-i\sin\theta}{(1-\cos\theta)^2+\sin^2\theta}=\frac{(1-\cos\theta)-i\sin\theta}{2(1-\cos\theta)}$$

$$=\frac{(1-\cos\theta)}{2(1-\cos\theta)}-i\frac{\sin\theta}{2(1-\cos\theta)}.$$

Therefore its real part $=\frac{1-\cos\theta}{2(1-\cos\theta)}=\frac{1}{2}$

Q.5 (2)

$$\sqrt{-7-24i}=x-iy$$

Squaring both sides, $-7-24i=x^2-y^2-i(2xy)$

Equating real and imaginary parts, we get

$x^2-y^2=-7$ and $2xy=24$

$\therefore\ x^2+y^2=\sqrt{49+576}=\sqrt{625}=25$

Q.6 (2)

Let $\sqrt{3-4i}=x+iy \Rightarrow 3-4i=x^2-y^2+2ixy$

$\Rightarrow x^2-y^2=3,\quad 2xy=-4$ ......(i)

$\Rightarrow (x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2$

$=(3)^2+(-4)^2=25$

$\Rightarrow x^2+y^2=5$ .....(ii)

From equation (i) and (ii) $x^2=4 \Rightarrow x=\pm 2$,

$y^2=1 \Rightarrow y=\pm 1$.

Hence the square root of $(3-4i)$ is $\pm(2-i)$

Q.7 (4)

$\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other if $\sin x=\cos x$ and $\cos 2x=\sin 2x$

or $\tan 2x=1 \Rightarrow 2x=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},........$ ……(i)

and $\tan 2x=1 \Rightarrow 2x=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},........$

or $x=\frac{\pi}{8},\frac{5\pi}{8},\frac{9\pi}{8}$ .... …..(ii)

There exists no value of x common in (i) and (ii). Therefore there is no value of for which the given complex numbers are conjugate.

Q.8 (3)

$$z=\frac{(2+i)^2}{3+i}=\frac{3+4i}{3+i}\times\frac{3-i}{3-i}=\frac{13}{10}+i\frac{9}{10}$$

Hence conjugate of $\frac{13}{10}+i\frac{9}{10}=\left(\frac{13}{10}-i\frac{9}{10}\right)$

Q.9 (3)

Given $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1 \Rightarrow \frac{z_1+z_2}{z_1-z_2}=\cos\theta+i\sin\theta$ (say)

$$\Rightarrow \frac{z_1}{z_2}=\frac{1+\cos\theta+i\sin\theta}{-1+\cos\theta+i\sin\theta}=-i\cot\frac{\theta}{2}$$

which is zero, if $\theta=n\pi(n\in I)$, and is otherwise purely imaginary.

Q.10 (1)

$$\left(\frac{3+2i}{3-2i}\right)=\left(\frac{3+2i}{3-2i}\right)\left(\frac{3+2i}{3+2i}\right)$$

$$= \frac{9-4+12i}{13} = \frac{5}{13} + i\left(\frac{12}{13}\right)$$

$$\text{Modulus} = \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = 1\,.$$

Q.11 (2)

We have $|z_1| = 1$ and z_2 be any complex number.

$$\Rightarrow \left|\frac{z_1 - z_2}{1 - z_1\bar{z}_2}\right| = \frac{|z_1 - z_2|}{\left|1 - \frac{\bar{z}_2}{\bar{z}_1}\right|}; \qquad \because z_1\bar{z}_1 = |z_1|^2$$

$$= \frac{|z_1 - z_2|}{|\bar{z}_1 - \bar{z}_2|}|\bar{z}_1|; \text{ Given that } \because |\bar{z}_1| = 1$$

$$= \frac{|z_1 - z_2|}{|\overline{z_1 - z_2}|} = \frac{|z_1 - z_2|}{|z_1 - z_2|} = 1\,.$$

Q.12 (3)

$|z| = 4$ and $\arg z = \frac{5\pi}{6} = 150^o$

Let $z = x + iy$, then $|z| = r = \sqrt{x^2 + y^2} = 4$

and $\theta = \frac{5\pi}{6} = 150^o$

$\therefore\ x = r\cos\theta = 4\cos 150^o = -2\sqrt{3}$

and $y = r\sin\theta = 4\ \sin 150^o = 4\frac{1}{2} = 2$

$\therefore z = x + iy = -2\sqrt{3} + 2i$

Trick : Since $\arg z = \frac{5\pi}{6} = 150^o$, here the complex number must lie in second quadrant, so (1) and (2) rejected. Also which satisfies (3) only.

Q.13 (1)

Let $z = \frac{1 + i\sqrt{3}}{1 + \sqrt{3}}$ $\therefore$ amp(z) or arg(z)

$$= \tan^{-1}\left[\frac{\sqrt{3}/(1+\sqrt{3})}{1/(1+\sqrt{3})}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Q.14 (4)

Let $z = 0 + ib$, where $b < 0$. Then z is represented by a point on OY' (negative direction of y – axis), therefore $arg(z) = -\frac{\pi}{2}$.

Q.15 (1)

Let $z = a + i0$, where $a < 0$. Then z is represented by a point on negative side of *x*-axis, therefore

$\arg(z) = \pi$

Q.16 (2)

$$\arg\left(\frac{13 - 5i}{4 - 9i}\right) = \arg(13 - 5i) - \arg(4 - 9i)$$

$$= -\tan^{-1}\left(\frac{5}{13}\right) + \tan^{-1}\frac{9}{4} = \frac{\pi}{4}$$

Q.17 (4)

$|z||\omega| = 1$ ......(i)

and $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2} \Rightarrow \frac{z}{\omega} = i\ \left|\frac{z}{\omega}\right| = 1$

.....(ii)

From equation (i) and (ii)

$|z| = |\omega| = 1$ and $\frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0;\ z\bar{\omega} + \bar{z}\omega = 0$

$\bar{z}\omega = -z\bar{\omega} = \frac{-z}{\omega}\bar{\omega}\omega\ ;\ \bar{z}\omega = -i|\omega|^2 = -i..$

Q.18 (2)

$$\frac{1 + 2i}{1 - (1 - i)^2} = \frac{1 + 2i}{1 - (1 - 1 - 2i)} = \frac{1 + 2i}{1 + 2i} = 1 + 0i$$

Modulus =1

Amplitude $\theta = \tan^{-1}\frac{0}{1} = 0$

Q.19 (4)

Given $z_1 = 1 + 2i$, $z_2 = 3 + 5i$ and $\bar{z}_2 = 3 - 5i$

$$\frac{\bar{z}_2 z_1}{z_2} = \frac{(3 - 5i)(1 + 2i)}{(3 + 5i)} = \frac{13 + i}{3 + 5i}$$

$$= \frac{13 + i}{3 + 5i} \times \frac{3 - 5i}{3 - 5i} = \frac{44 - 62i}{34}$$

Then $\text{Re}\left(\frac{\bar{z}_2 z_1}{z_2}\right) = \frac{44}{34} = \frac{22}{17}$

Q.20 (2)

Given that $z = \left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)^5$

$$= \left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right]^5 + \left[\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right]^5$$

$$= \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + \cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}\,.$$

Hence Im (z) = 0.

Q.21 (1)

Given that $z = \frac{\sqrt{3} + i}{2} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$$\Rightarrow\ iz = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = \omega$$

Now

$$z^{69}=z^{4(17)}z=(iz)^{4(17)}z=(\omega)^{68}z,\ \ (\because i^{4n}=1)$$

$$=\frac{\omega^{69}}{i}=\frac{(\omega^3)^{23}}{i}=\frac{1}{i}=-i$$

Aliter : $z=\frac{\sqrt{3}}{2}+i\frac{1}{2}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

$$\Rightarrow z^{69}=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{69}=\cos\frac{69\pi}{6}+i\sin\frac{69\pi}{6}$$

$$=\cos\left(11\pi+\frac{\pi}{2}\right)+i\sin\left(11\pi+\frac{\pi}{2}\right)=0+i(-1)=-i\ .$$

Q.22 (3)

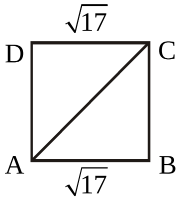
$$(1-\omega+\omega^2)(1-\omega^2+\omega)^6=(-2\omega)(-2\omega^2)^6=-128\omega.$$

Q.23 (3)

Given the vertices of quadrilateral

$A(1+2i), B(-3+i), C(-2-3i)$ and $D(2-2i)$

Now, $AB=\sqrt{16+1}=\sqrt{17}, BC=\sqrt{16+1}=\sqrt{17}$



$$CD=\sqrt{16+1}=\sqrt{17}, DA=\sqrt{16+1}=\sqrt{17}$$

$$AC=\sqrt{9+25}=\sqrt{34}, BD=\sqrt{25+9}=\sqrt{34}$$

$$\frac{1}{2}d^2=\left(\sqrt{17}\right)^2$$

$d=\sqrt{34}$

Hence it is a square.

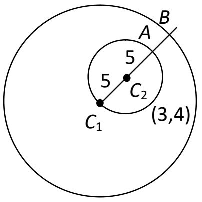
Q.24 (2)

The two circles are $C_1(0,0), r_1=12$, $C_2(3,4), r_2=5$ and it passes through origin, the centre of C_1.

$C_1C_2=5<r_1-r_2=7$

Hence circle C_2

lies inside circle .



Therefore minimum distance between them is

$$AB=C_1B-C_1A=r_1-2r_2=12-10=2.$$

Q.25 (2)

Let z = x + iy ; z + iz = (x – y) + i(x + y)

and iz = – y + ix

If A denotes the area of the triangle formed by

$z, z+iz$ and iz , then $A=\frac{1}{2}\begin{vmatrix} x & y & 1\\ x-y & x+y & 1\\ -y & x & 1\end{vmatrix}$

Applying transformation $R_2\to R_2-R_1-R_3$, we get

$$A=\frac{1}{2}\begin{vmatrix} x & y & 1\\ 0 & 0 & -1\\ -y & x & 0\end{vmatrix}=\frac{1}{2}(x^2+y^2)=\frac{1}{2}|z|^2$$

Q.26 (2)

$z=(x+iy)\Rightarrow z^2=x^2-y^2+2ixy$

$\Rightarrow \text{Re}(z^2)=1\Rightarrow x^2-y^2=1$, which is a hyperbola.

Q.27 (4)

$$\arg\{(x-a)+iy\}=\frac{\pi}{4}\Rightarrow \tan^{-1}\left(\frac{y}{x-a}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{y}{x-a}=\tan\frac{\pi}{4}=1\Rightarrow x-a=y$$

Q.28 (3)

Given $|8+z|+|z-8|=16$. Locus is a straight line

JEE MAIN

OBJECTIVE QUESTIONS

Q.1 (2)

$$\left(\frac{1+i}{1-i}\right)^n=\left(\frac{e^{i\pi/4}}{e^{-i\pi/4}}\right)^n=e^{i\frac{n\pi}{2}}\Rightarrow n=2$$

Q.2 (2)

z = z + iy

$\Rightarrow z^2=x^2-y^2+2\,ixy\Rightarrow \text{Re}(z^2)=x^2-y^2,$

$|z|=\sqrt{x^2+y^2}$

$$\Rightarrow x^2-y^2=0,\quad x^2+y^2=3\quad\Rightarrow x^2=y^2=\frac{3}{2}$$

$$\Rightarrow x=\pm\sqrt{\frac{3}{2}}, y=\pm\sqrt{\frac{3}{2}}$$

Q.3 (1)

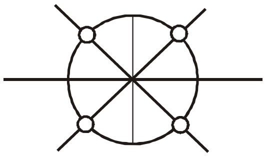
$(b+ia)^5=i(-ib+a)^5=i(\alpha-i\beta)=\beta+i\alpha$

Q.4 (1)

$\text{Re}(z^2)=0$

$x^2-y^2=0$

$x=\pm y$



Four solution

Q.5 (1)

|z – 5i| = |z + 5i| implies perpendicular bisector of line joining 5i & – 5i which is real axis.

Q.6 (4)

$z = (3p - 7q) + (7p + 3q)\,i$

$\because$ z is purely imaginary $3p = 7q$

$p = 7 \qquad q = 3$ for minimum value of $|z|^2$

$|z|^2 = 58 \times 58 = 3364$

Q.7 (1)

$|(2 + i)(2 + 2i)(2 + 3i) \,.....\, (2 + ni)| = |x + iy|$

$\Rightarrow$ $5.8.13......... (4 + n^2) = (x^2 + y^2)$

Q.8 (3)

Let α be the real root

$\alpha^2 - (3 + i)\alpha + m + 2i = 0$

$(\alpha^2 - 3\alpha + m) + i(2 - \alpha) = 0$

$\therefore$ $\alpha = 2$ (real root)

$\therefore$ $4 - 6 + m = 0 \Rightarrow m = 2$

Product of the roots = 2(1 + i) with one root as 2

non real root = 1 + i, addivitve inverse is – 1 – i **Ans**

Q.9 (3)

We have, $\arg\left(\frac{z_1}{z_2}\right) = \pi$

$\Rightarrow \arg(z_1) - \arg(z_2) = \pi \quad \Rightarrow \arg(z_1) = \arg(z_2) + \pi$

Let $\arg(z_2) = \theta$. Then $\arg(z_1) = \pi + \theta$.

$\therefore$ $z_1 = |z_1|\,[\cos(\pi + \theta) + i\sin(\pi + \theta)] = |z_1|(-\cos\theta - i\sin\theta)$ and

$z_2 = |z_2|(\cos\theta + i\sin\theta) = |z_1|(\cos\theta + i\sin\theta)$

$(\therefore |z_1| = |z_2|) = -z_1$

$\Rightarrow$ $z_1 + z_2 = 0$

Q.10 (1)

$T_n = \left(\frac{\sqrt{3} + i}{2}\right)^n \qquad \Rightarrow |T_n| = 1$

Q.11 (4)

$z^{1/3} = a - ib$

$\Rightarrow$ $z = (a - ib)^3$

$\Rightarrow$ $x + iy = (a^3 - 3ab^2) + i(b^3 - 3a^2b)$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2 \qquad \frac{y}{b} = b^2 - 3a^2$

$\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2) \qquad k = 4$

Q.12 (4)

$|\,iz + 3 - 4i| \le |iz| + |3 - 4i| = |z| + 5 \le 9$

Q.13 (2)

$x^2 + i(a - 1)\,x + 5 = 0$ roots of this equation are p + iq, p – iq where $p, q \in R$

$p + iq + p - iq = i(a - 1)$

$\Rightarrow$ $2p + i(a - 1) = 0$

hence a = 1

Q.14 (4)

$\text{Amp}\left(\frac{z-1}{z+3}\right) = 0 \qquad \Rightarrow \text{Im}\left(\frac{z-1}{z+3}\right) = 0$

$\Rightarrow$ y = 0, Hence (x – 1) : y = ∞ (does not exist)

Q.15 (3)

The expression is the sum of the distance of z from the two points 1 – 2i and – 3 + 4i. The minimum value is the distance between these two points = $\sqrt{4^2 + 6^2}$

= $2\sqrt{13}$ **Ans.**

Q.16 (1)

$z = \frac{1 - i}{1 + i}\,\bar{z} \qquad \Rightarrow z = -i\,\bar{z}$

$\Rightarrow x + iy = -y - ix \qquad \Rightarrow x = -y$

so $z = x - ix$, $x \in R$ $\qquad \Rightarrow x(1 - i)$

or $t(1 - i)$, $t \in R$.

Q.17 (4)

$$\frac{\bar{z}_1}{z_2} = \frac{r_1 e^{-i\theta_1}}{r_2 e^{i\theta_2}} = 2e^{-(\theta_1 + \theta_2)\,i} = 2e^{-i\frac{3\pi}{2}} = 2i$$

Q.18 (2)

Given that, $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 + (-1)| = |z^2| + |-1|$

It shows that the origin, –1 and z^2 lies on a line and z^2 and –1 lies on one side of the origin, therefore z^2 is a negative number. Hence, z will be purely imaginary. So, we can say that z lies on y-axis.

Q.19 (4)

$|z - 4| < |z - 2| \qquad \Rightarrow$

$z\bar{z} + 16 - 4(z + \bar{z}) < z\bar{z} + 4 - 2(z + \bar{z})$

$\Rightarrow$ $12 < 2(z + \bar{z}) \qquad \Rightarrow 12 < 4\text{Re}(z)$

$\Rightarrow$ $3 < \text{Re}(z)$

Q.20 (1)

New $z = \frac{3}{2}(-4 + 5i)e^{i\pi} = 6 - \frac{15}{2}i$

Q.21 (3)

Let $Z = \cos\theta + i\sin\theta = e^{i\theta}$ and so on

$\Rightarrow$ $e^{i\theta}\,.\,e^{i\,2\theta}\ \ldots\ldots\ldots\ldots\ldots\ e^{i\,n\theta} = 1$

$\Rightarrow$ $e^{i\left(\frac{n(n+1)}{2}\right)\theta} = e^{2m\pi + 0}$

$\Rightarrow$ $\theta = \frac{4m\pi}{n(n+1)}$; $\qquad m \in z$

Q.22 (1)

$\left(\frac{e^{i\alpha}}{e^{-i\alpha}}\right)^n - \left(\frac{e^{in\alpha}}{e^{-in\alpha}}\right) \qquad \Rightarrow e^{i2n\alpha} - e^{i2n\alpha} = 0$

Q.23 (3)

We have

$\alpha^{2n} + 2^n\alpha^n + 2^{2n} = 2^{2n}\left[\left(\frac{\alpha}{2}\right)^{2n} + \left(\frac{\alpha}{2}\right)^n + 1\right]$

$= 2^{2n}(\omega^{2n}+\omega^{n}+1)\left[\because \frac{\alpha}{2}=\frac{-1}{2}+\frac{i\sqrt{3}}{2} \text{ and } \left(\frac{\alpha}{2}\right)^{3}=1\right]$

$= 2^{2n}(0) = 0$ (Since 3 does not divide n)

Q.24 (4)

We have,

$|z| = |\omega|$ and $\arg z = \pi - \arg \omega$

Let $\omega = re^{i\theta}$. Then $z = re^{i(\pi-\theta)}$

$\Rightarrow z = re^{i\pi}e^{-i\theta} = (re^{-i\theta})(\cos\pi + i\sin\pi) = \overline{\omega}(-1) = -\overline{\omega}$

Q.25 (1)

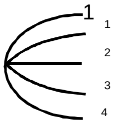
Given, $\Delta = \begin{vmatrix} 1 & \omega^{n} & \omega^{2n} \\ \omega^{n} & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^{n} \end{vmatrix} = 1(\omega^{3n}-1) - \omega^{n}(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^{n} - \omega^{4n})$

$= 1(1-1) - 0 + \omega^{2n}(\omega^{n} - \omega^{n}) = 0$

Q.26 (1)

$\alpha = 1^{1/5}$

consider $x^5 - 1 = 0$



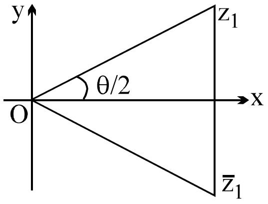
so $2^{\left|1+\alpha+\alpha^{2}+\frac{\alpha^{3}}{\alpha^{5}}-\frac{\alpha^{4}}{\alpha^{5}}\right|} = 2^{|1+\alpha+\alpha^{2}+\alpha^{3}-\alpha^{4}|}$

$= 2^{|-\alpha^{4}-\alpha^{4}|} = 4^{|\alpha^{4}|} = 4$

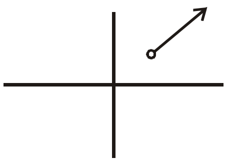
Q.27 (1)

$\frac{y}{x} = \tan\frac{\theta}{2} = \sqrt{2} - 1 = \tan\frac{\pi}{8}$

$= \frac{\theta}{2} = \frac{\pi}{8} \Rightarrow \theta = 45° \Rightarrow n = \frac{360°}{45°} = 8$

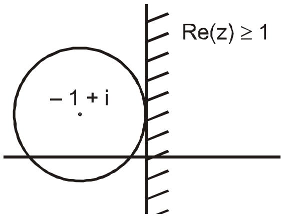


Q.28 (1)



Q.29 (2)

$|z + 1 - i| = 2$



Only one solution

Q.30 (1)

Obvious (1) $|z| \le 1$ is region interior of circle of radius 1 unit and centre (0, 0)

and arg. of given region is $-\pi/2 \le \arg z \le \pi/2$

Q.31 (3)

$|z-1|^2 + |z+1|^2 = 2 \Rightarrow$

$z\overline{z} + 1 - (z+\overline{z}) + z\overline{z} + 1 + (z+\overline{z}) = 2$

$\Rightarrow z\overline{z} = 0 \qquad \Rightarrow |z| = 0$

$\Rightarrow z = 0 \qquad \Rightarrow x = o, y = 0$

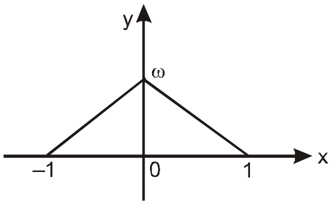
Q.32 (3)

A(1, 0) and B(– 1, 0) are two fixed point at a distance $\sqrt{2}$ & P(z) moves so that PA + PB ≤ 4. Hence locus of P is ellipse.

Q.33 (1)

$\omega = \frac{z-1}{z+1}$

$\Rightarrow \omega z + \omega = z - 1$



$\Rightarrow (\omega - 1)z = -1 - \omega \qquad \Rightarrow z = \frac{1+\omega}{1-\omega}$

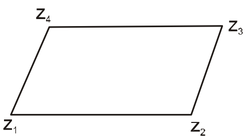
Now $|z| = 1 \qquad \Rightarrow \left|\frac{1+\omega}{1-\omega}\right| = 1$

$\Rightarrow |\omega - (-1)| = |\omega - 1|$

$\Rightarrow$ ω lies on the perpendicular bisector of the segment joining – 1 and 1.

Thus, ω lies on the imaginary axis.

Q.34 (2)



Mid points of diagonals are equal

$\frac{z_1+z_3}{2} = \frac{z_2+z_4}{2}$

$\Rightarrow z_1 + z_3 = z_2 + z_4$

Q.35 (4)

Triangle formed by the points A(z), B(ω z) and C(z + ωz) is clearly isosules with angle between the equal sides being $\frac{2\pi}{3}$.

$$\Delta_{ABC} = \frac{1}{2}r^2\,\frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$\Rightarrow$ r = 4 units

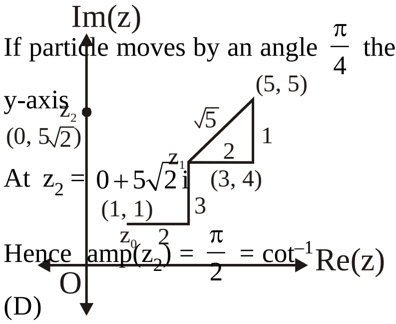
JEE ADVANCED
OBJECTIVE QUESATIONS

Q.1 (B)

Clearly $z_1 = 3 + 4i$

After moving by $\sqrt{5}$ distance in direction of $2\hat{i}+\hat{j}$, particle will

reach at point $\left(5\hat{i}+5\hat{j}\right)$



If particle moves by an angle $\frac{\pi}{4}$ then it will reach at y-axis

At $z_2 = 0+5\sqrt{2}\,i$

Hence amp(z_2) = $\frac{\pi}{2}$ = $\cot^{-1}$

Q.2 (D)

$$z = \frac{\pi}{4}(1+i)^4\left[\frac{1+\pi+\pi+1}{(\sqrt{\pi}+i)(1+\sqrt{\pi}\,i)}\right] = \frac{\pi}{4}(1+i)^4\;\frac{2}{i} =$$

$$\frac{\pi}{2}\frac{(1+i)^4}{i} = \frac{\pi}{2}\,\frac{4e^{i\pi}}{e^{i\pi/2}} = 2\pi e^{i\pi/2}$$

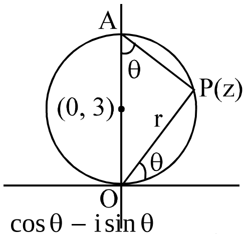
$|z| = 2\pi$ amp (z) = $\frac{\pi}{2}$

$$\left(\frac{|z|}{\text{amp}(z)}\right) = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Q.3 (C)

z = r (cos θ + i sin θ) now r = OA sin θ = 6 sin θ

$$z = 6\sin\theta\,(\cos\theta + i\sin\theta) \qquad \frac{6}{z} = \frac{1}{\sin\theta\,(\cos\theta + i\sin\theta)}$$



$$= \frac{\cos\theta - i\sin\theta}{\sin\theta} = -i + \cot\theta \Rightarrow \cot\theta - \frac{6}{z} = i$$

Q.4 (B)

$4z_2z_3 + 9z_3z_1 + 16z_1z_2 = \bar{z}_1 z_1 z_2 z_3 + \bar{z}_2 z_2 z_3 z_1 + \bar{z}_3 z_1 z_2 z_3$

$= (\bar{z}_1 + \bar{z}_2 + \bar{z}_3)(z_1z_2z_3) = \overline{(z_1+z_2+z_3)}\,(z_1z_2z_3) = |z_1||z_2||z_3|\,|z_1+z_2+z_3|$

So absolute value = 2. 2. 3. 4 = 48

Q.5 (B)

$$E = \frac{1}{z_1-1}+\frac{1}{z_2-1}+\ldots..+\frac{1}{z_{50}-1},$$ where $z_1, z_2, \ldots., z_{50}$ are the roots of the equation $z^{51} - 1 = 0$ other than 1.

= – 25 +

$$\left(\frac{1}{2}+\frac{1}{z_1-1}\right)+\left(\frac{1}{2}+\frac{1}{z_2-1}\right)+\ldots\ldots+\left(\frac{1}{2}+\frac{1}{z_{50}-1}\right)$$

Note that (1st + last) and (2nd + 2nd last) will vanish using $z_r = z^r$ and $z^{51} = 1$

Alternatively:

Let $1 + z + z^2 + \ldots\ldots + z^{50} = (z - z_1)(z - z_2)(z - z_{50})$ differentiate both sides w.r.t. z after taking logarithm on both the sides.

$$\frac{1+2z+3z^2+\ldots.+50z^{49}}{1+z+z^2+\ldots..+z^{50}} =$$

$$\frac{1}{z-z_1}+\frac{1}{z-z_2}+\ldots\ldots+\frac{1}{z-z_{50}}.$$ Now put z = 1

we get, $$\frac{50\cdot 51}{2\cdot 51} = -\left[\frac{1}{z_1-1}+\frac{1}{z_2-1}+\ldots\ldots+\frac{1}{z_{50}-1}\right]$$

$$\therefore \sum\frac{1}{z_r-1} = -25 \quad \text{Ans.}$$

Q.6 (B)

Let $x\,i$ be the root where $x \neq 0$ and $x \in R$ (as if x = 0 satisfies then $a_4 = 0$ which contradicts)

$x^4 - a_1x^3 i - a_2x^2 + a_3 x i + a_4 = 0$

$x^4 - a_2x^2 + a_4 = 0$ ....(1)

and

$a_1x^3 - a_3x = 0$ ....(2)

From equation (2) : $a_1x^2 - a_3 = 0 \Rightarrow x^2 = a_3/a_1$.(as $x \neq 0$)

Putting the value of x^2 in equation .....(1)

$$\frac{a_3^2}{a_1^2} - \frac{a_2\,a_3}{a_1} + a_4 = 0 \text{ or } a_3^2 + a_4a_1^2$$

$$= a_1a_2a_3 \text{ or } \frac{a_3}{a_1\,a_2} + \frac{a_1a_4}{a_2\,a_3} = 1 \text{ (dividing by } a_1a_2a_3)$$

Q.7 (C)

Letz = cos x + i sin x, x ∈ [0, 2π]. Then

$$1 = \left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right| = \frac{|z^2+\bar{z}^2|}{|z|^2}$$ = |cos 2x + i sin 2x + cos 2x – i sin 2x| = 2|cos 2x|

$\Rightarrow$ cos 2x = ± 1/2

Now,

$$\cos 2x = 1/2 \Rightarrow \quad x_1 = \frac{\pi}{6},\ x_2 = \frac{5\pi}{6},$$

$$x_3 = \frac{7\pi}{6},\ x_4 = \frac{11\pi}{6}$$

$$\cos 2x = -\frac{1}{2} \Rightarrow \quad x_5 = \frac{\pi}{3},\ x_6 = \frac{2\pi}{3},\ x_7 = \frac{4\pi}{3},$$

$$x_8 = \frac{5\pi}{3}$$

Q.8 (B)

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$.

Then $x_1^2 = (x_1 - 2)^2 + y_1^2$ and $x_2^2 = (x_2 - 2)^2 + y_2^2$

Therefore

$4x_1 = y_1^2 + 4$ and $4x_2 = y_2^2 + 4$

On subtraction we get

$4(x_1 - x_2) = y_1^2 - y_2^2 = (y_1 + y_2)(y_1 - y_2)$

$$\text{Hence} \quad y_1 + y_2 = \frac{4(x_1 - x_2)}{y_1 - y_2} \quad \ldots\text{(i)}$$

Also $\arg(z_1 - z_2) = \frac{\pi}{3}$. Therefore

$$\tan\frac{\pi}{3} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\Rightarrow \sqrt{3} = \frac{y_1 - y_2}{x_1 - x_2} \qquad \ldots\text{(ii)}$$

From (i) and (ii), we have

$$\text{Im}(z_1 + z_2) = y_1 + y_2 = \frac{4}{\sqrt{3}}$$

Q.9 (A)

$$\text{We have} \quad \frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|} = k \text{ (let)}$$

$$\Rightarrow \frac{9}{|z_2 - z_3|^2} = \frac{16}{|z_3 - z_1|^2} = \frac{25}{|z_1 - z_2|^2} = k^2$$

$$\text{Now} \quad \frac{9}{|z_2 - z_3|^2} = k^2 \Rightarrow \frac{9}{z_2 - z_3} =$$

$$k^2(\bar{z}_2 - \bar{z}_3) \qquad \text{....(1)}$$

[As $|z|^2 = z\bar{z}$]

$$\text{|||ly} \ \frac{16}{|z_3 - z_1|^2} = k^2 \qquad \Rightarrow \frac{16}{z_3 - z_1} =$$

$$k^2(\bar{z}_3 - \bar{z}_1) \qquad \text{....(2)}$$

$$\text{|||ly} \ \frac{25}{|z_1 - z_2|^2} = k^2 \qquad \Rightarrow \frac{25}{z_1 - z_2} =$$

$$k^2(\bar{z}_1 - \bar{z}_2) \qquad \text{....(3)}$$

$\therefore$ On adding (1), (2) and (3), we get

$$\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2} = k^2$$

$$(\bar{z}_2 - \bar{z}_3 + \bar{z}_3 - \bar{z}_1 + \bar{z}_1 - \bar{z}_2) = 0$$

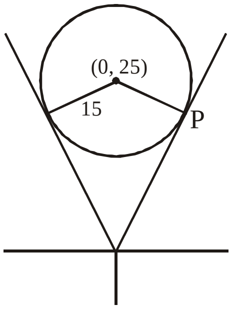
Q.10 (D)

The required complex number is point of contact P as shown in the figure. C(0, 25) is the centre of the circle and radius is 15.

$$\text{Now } |z| = OP = \sqrt{OC^2 - PC^2}$$

$$= \sqrt{625 - 225} = 20$$

amp (z) = θ = ∠ XOP = ∠ OCP



$$\therefore \cos\theta = \frac{PC}{OC} = \frac{15}{25} = \frac{3}{5}$$

$$\text{and } \sin\theta = \frac{OP}{OC} = \frac{20}{25} = \frac{4}{5}$$

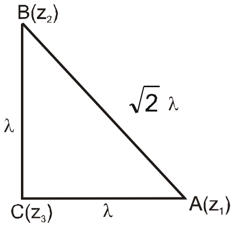
$$\therefore z = 20\left(\frac{3}{5} + \frac{4}{5}i\right) = 12 + 16i.$$

Q.11 (B)

Apply Rotation

$$\sqrt{2}\,(z_3 - z_1) = (z_2 - z_1)\,e^{i\pi/4}$$

$$(z_1 - z_2) = \sqrt{2}\,(z_3 - z_2)e^{i\pi/4}$$



$$\frac{\sqrt{2}(z_3 - z_1)}{(z_1 - z_2)} = \frac{(z_2 - z_1)}{\sqrt{2}(z_3 - z_2)}$$

$$2(z_3 - z_2)(z_1 - z_3) = (z_1 - z_2)^2$$

Q.12 (D)

Let $S = 1(\alpha_1 + \alpha_{2008}) + 2(\alpha_2 + \alpha_{2007}) + 3(\alpha_3 + \alpha_{2006})$ + ... + $2008(\alpha_{2008} + \alpha_1)$ ....(1)

Also $S = 2008(\alpha_{2008} + \alpha_1) + 2007(\alpha_2 + \alpha_{2007}) + \ldots\ldots\ldots + 2(\alpha_2 + \alpha_{2007}) + 1(\alpha_1 + \alpha_{2008})$ ....(2)

(writing in reverse order)

$\therefore$ On adding (1) and (2), we get

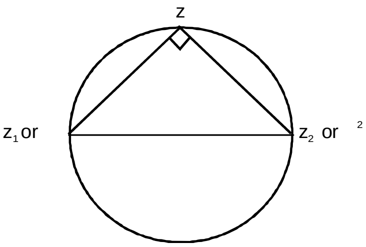
$$2S = 2009[2(\alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots.\alpha_{2008})]$$
$$2S = 2009[2\,(\underbrace{1 + \alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots.\alpha_{2008}}_{\text{zero}} - 1)]$$

Hence $S = -2009$ **Ans.**

Note that (α_1 and α_{2008}), (α_2 and α_{2007}), (α_3 and α_{2006}), ..., (α_{1004} and α_{1005}) are conjugate of each other.

Q.13 (B)

$\because$ Circle



so by pythogorous theorem $\lambda = |w - w^2|^2 = \left|\sqrt{3}\right|^2$

$= 3$

Q.14 (C)

$x = a + b + c$

$y = w(a + bw + cw^2)$

$z = w^2(a + bw^2 + cw)$.

$xyz = (a + b + c)(a + bw + cw^2)(a + bw^2 + cw)$

$= a^3 + b^3 + c^3 - 3abc$

Q.15 (A)

$\because$ Roots are ω and ω^2

$$\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega^2 + \frac{1}{\omega^2}\right)^2 + \left(\omega^3 + \frac{1}{\omega^3}\right)^2 \ldots\ldots \left(\omega^{27} + \frac{1}{\omega^{17}}\right)^2$$

there are 9 term which have ω^{3p}. so sum $9 \times 4 = 36$

there are 18 term which not have ω^{3p}

so sum is $= 18$. Total sum $= 18 + 36 = 54$

Q.16 (B)

We have $(k + 1)(k\omega + 1)(k\omega^2 + 1)$

$= (k + 1)(k^2 - k + 1) = k^3 + 1$

Therefore $E = \sum_{k=1}^{n}(k^3 + 1)$

$= \sum_{k=1}^{n} k^3 + n = \dfrac{n^2(n+1)^2}{4} + n$

Q.17 (B)

From the hypothesis we have

$z = \dfrac{\sqrt{3}}{2} - \dfrac{i}{2} = i\left(-\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}\right) = i\omega$

where $\omega = \left(-\dfrac{1}{2}\right) - \left(\dfrac{i\sqrt{3}}{2}\right)$ which is a cube root of unity. Now $z^{95} = (i\omega)^{95} = -i\omega^2$ (since $\omega^3 = 1$) and $i^{67} = i^3 = -i$.

Therefore, $z^{95} + i^{67} = -i(1 + \omega^2) = (-i)(-\omega) = i\omega$

$(z^{95} + i^{67})^{94} = (i\omega)^{94} = i^2\omega = -\omega$

Now $-\omega = z^n = (i\omega)^n$

$\Rightarrow$ $i^n \cdot \omega^{n-1} = -1$

$\Rightarrow$ $n = 2, 6, 10, 14, \ldots\ldots$ and $n - 1 = 3, 6, 9, \ldots\ldots$

Therefore $n = 10$ is the required least positive integer.

Q.18 (B)

Let $z = 1 + i = \sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

Thefore $z^{3/4} = 2^{3/8}$

$$= \left[\cos\left(2k\pi + \frac{\pi}{4}\right)\frac{3}{4} + i\sin\left(2k\pi + \frac{\pi}{4}\right)\frac{3}{4}\right]$$

for $k = 0, 1, 2, 3$. The product of the values of this is equal to

$$2^{3/2}\left[\text{cis}\left(\frac{\pi}{4} + \frac{9\pi}{4} + \frac{17\pi}{4} + \frac{25\pi}{4}\right)\frac{3}{4}\right]$$

$$= 2^{3/2}\ \text{cis}\left(\frac{52\pi}{4}\cdot\frac{3}{4}\right) = 2^{3/2}\ \text{cis}\,\frac{39\pi}{4}$$

$$= 2^{3/2}\ \text{cis}\left(9\pi + \frac{3\pi}{4}\right) = 2^{3/2}\ \text{cis}\left(10\pi - \frac{3\pi}{4}\right) = 2^{3/}$$

$$2\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right] = 2\,(1 - i)$$

Q.19 (D)

Given $|\,z - |z + 1|\,|^2 = |\,z + |z - 1|\,|^2$

$\therefore$ $\left(z - |z+1|\right)\left(\bar{z} - |z+1|\right) =$

$\left(z + |z-1|\right)\left(\bar{z} + |z-1|\right)$

$z\bar{z} - z|z+1| - \bar{z}|z+1| + |z+1|^2 =$

$z\bar{z} + z|z-1| + \bar{z}|z-1| + |z-1|^2$

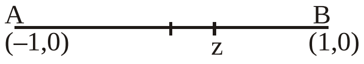
$|z+1|^2 - |z-1|^2 = (z + \bar{z})\left[|z-1| + |z+1|\right]$

$(z+1)(\bar{z}+1) - (z-1)(\bar{z}-1) = (z + \bar{z})\left[|z-1| + |z+1|\right]$

$\left(z\bar{z} + z + \bar{z} + 1\right) - \left(z\bar{z} - z - \bar{z} + 1\right) = \left(z + \bar{z}\right)\left[|z-1| + |z+1|\right]$

$2\left(z + \bar{z}\right) = \left(z + \bar{z}\right)\left[|z+1| + |z-1|\right]$

$\left(z + \bar{z}\right)\left[|z+1| + |z-1| - 2\right] = 0$



$\Rightarrow$ either $z + \bar{z} = 0$ $\Rightarrow$ z is purely imaginary

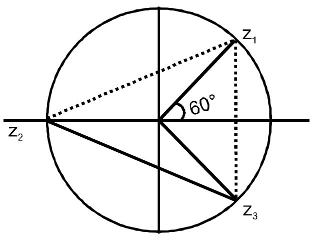
⇒ z lies on y axis ⇒ x = 0

or |z + 1| + |z – 1| = 2

⇒ z lie on the line segment joining (–1, 0) and (1, 0)

Q.20 (C)

$z_1 = 1 + i\sqrt{3} = 2e^{\frac{i\pi}{3}}$



$z_2 = 2e^{i\pi} = -2$

$\Rightarrow z_3 = 2\,e^{-\frac{\pi}{3}i} = 1 - i\sqrt{3}$

Q.21 (D)

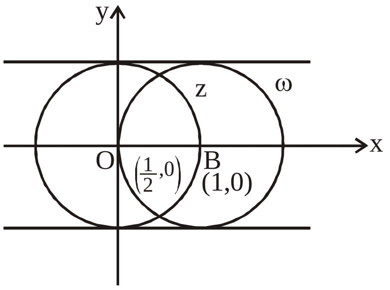
$E = |2z - 1|^2 + |2\omega - 1|^2$

$$E = \left[\left|z-\frac{1}{2}\right|^2 + \left|\omega-\frac{1}{2}\right|^2\right]$$

$\left|z-\frac{1}{2}\right|$ distance of z tan $\left(\frac{1}{2}, 0\right)$ and $\left|\omega-\frac{1}{2}\right|$

distance of ω from $\left(\frac{1}{2}, 0\right)$.

$$E_{max} = 4\left[\left(-1, -\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2\right] = 4\left[\frac{9}{4}+\frac{9}{4}\right] = 18$$



$$E_{min} = 4\left[\left(1-\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2\right] = 4\left[\frac{1}{4}+\frac{1}{4}\right] = 2$$

rajected E = [2, 18]

Alternatively :

$(2z - 1)\,(2\overline{z}-1) + (2\omega - 1)\,(2\overline{\omega}-1)$

$\left[4|z|^2 - 2(z+\overline{z})+1\right] + \left[4|\omega|^2 - 2(\omega+\overline{\omega})+1\right]$

$E = 6 - 4R + z + 4\,\omega\overline{\omega} - 4\,\text{Re}\omega$

Now $|\omega - 1|^2 = 1 \Rightarrow (\omega - 1)(\overline{\omega} - 1) = 1$

$\omega\overline{\omega} - (\omega + \overline{\omega}) = 0$

$\omega\overline{\omega} = \omega + \overline{\omega}$

$\omega\overline{\omega} = 2\,\text{Re}\,\omega$

Hence E = 6 – 4Rez + 4 Reω

= 6 + 4 (Re ω – Re z)

= $6 + 4\,(x_2 - x_1)$

$E_{max} = 6 + 4\,(2 + 1) = 18$

$E_{min} = 6 + 4\,(0 - 1) = 2 \Rightarrow$ say [2, 18]

JEE-ADVANCED MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (BD)

$$i^n = \left(e^{i\frac{\pi}{2}}\right)^{\frac{2n}{2}} = \left(e^{i\frac{\pi}{4}}\right)^{2n} = \frac{(1+i)^{2n}}{2^n}$$

$$i^{-n} = \frac{(1-i)^{+2n}}{2^{+n}}$$

$$\therefore \text{ G.E.} = \frac{(1+i)^{2n}}{2^n} + \frac{(1-i)^{2n}}{2^n}$$

Q.2 (CD)

Let z = x + iy

from 2nd equation x = 6 put in (1)

3 | (x – 12) + yi | = 5 | x + (y – 8)i |

$9\,[36 + y^2] = 25\,[36 + (y - 8)^2]$

(substituting x = 6)

$9 \cdot 36 + 9y^2 = 25 \cdot 36 + 25[y^2 + 64 - 16y]$

$16y^2 - 25 \cdot 16\,y + 36 \cdot 16 + 25 \cdot 64 = 0$

$y^2 - 25y + 36 + 100 = 0$

$y^2 - 25y + 136 = 0$

(y – 17)(y – 8) = 0

then

y = 17 or y = 8 ⇒ (C), (D)

Q.3 (ABCD)

$z = i^n$

Now principal argument of z can be 0,

π, π/2, –π/2

Q.4 (ABC)

(A)Let real root be α, then $\alpha^2 + (p + ip')\,\alpha + q + iq' = 0 \Rightarrow \alpha^2 + p\alpha + q = 0$ & $p'\alpha + q' = 0$

$$\Rightarrow \frac{q'^2}{p'^2} + p\left(\frac{-q'}{p'}\right) + q = 0 \Rightarrow q'^2 - pp'q' + qp'^2 = 0$$

(B)equal roots ⇒ D = 0

$\Rightarrow (p + ip')^2 - 4.(q + iq') = 0$

$\Rightarrow p^2 - p'^2 = 4q$ & $pp' = 2q'$

then equal roots are $= -\frac{p+ip'}{2}, -\frac{p+ip'}{2}$; the roots will be complex.

Q.5 (B, C)

amp $(z_1z_2) = 0 \Rightarrow$ amp z_1 + amp z_2 = 0

$\therefore$ amp z_1 = –amp z_2 = amp $\bar{z}_2$

Since $|z_1| = |z_2|$, we get $|z_1| = |\bar{z}_2|$. So , $z_1 = \bar{z}_2$. Also

, $z_1z_2 = \bar{z}_2 z_2 = |z_2|^2$

= 1 because $|z_2| = 1$.

Q.6 (BD)

$\theta_1 - \pi/4 = \theta_2 + 2k\pi$ and $\theta_1 + \theta_2 = \pi/2 + 2\lambda\pi$

Q.7 (BCD)

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$

$\Rightarrow \dfrac{z_1}{z_2} = -\dfrac{\bar{z}_1}{\bar{z}_2}$

$\dfrac{z_1}{z_2} + \overline{\left(\dfrac{z_1}{z_2}\right)} = 0$

$\Rightarrow \dfrac{z_1}{z_2}$ is purely imaginary

so amp$\left(\dfrac{z_1}{z_2}\right)$ is may be $\dfrac{\pi}{2}$ or $-\dfrac{\pi}{2}$

Q.8 (ABD)

Given that z and ω are two complex numbers. To prove

$|z|^2 w - |w|^2 z = z - w \Leftrightarrow z = w$ or $z\bar{w} = 1$

First let us consider

$|z|^2 w - |w|^2 z = z - w$ ....(1)

$\Rightarrow z(1 + |w|^2) = w(1 + |z|^2)$

$\Rightarrow \dfrac{z}{w} = \dfrac{1+|z|^2}{1+|w|^2}$

$\Rightarrow \overline{\left(\dfrac{z}{w}\right)} = \dfrac{z}{w} \Rightarrow \dfrac{\bar{z}}{\bar{w}} = \dfrac{z}{w}$

$\Rightarrow \bar{z}w = z\bar{w}$ ....(2)

Again from Eq. (1),

$z\bar{z}w - w\bar{w}z = z - w$

$z(\bar{z}w - 1) - w(\bar{w}z - 1) = 0$

$z(z\bar{w} - 1) - w(z\bar{w} - 1) = 0$

[Using Eq. (2)]

$\Rightarrow (z\bar{w} - 1)(z - w) = 0$

$\Rightarrow z\bar{w} = 1$ or $z = w$

Q.9 (ABCD)

$|z_1 + z_2|^2 = (z_1 + z_2)\left(\bar{z}_1 + \bar{z}_2\right) = |z_1|^2 + |z_2|^2$

$= z_1\bar{z}_2 + \bar{z}_1 z_2$

and $|z_1 - z_2|^2 = (z_1 - z_2)$

$\left(\bar{z}_1 - \bar{z}_2\right)$

Therefore

$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

(A is true)

$|z_1 + z_2|^2 - |z_1 - z_2|^2 = 2\left(z_1\bar{z}_2 + \bar{z}_1 z_1\right)$

(D is true)

Now, $\left(\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right| \right)^2$

$= \left|z_1 + \sqrt{z_1^2 - z_2^2}\right|^2 + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right|^2 +$

$2\left|z_1^2 - (z_1^2 - z_2^2)\right|$

$= 2\left(|z_1|^2 + |z_1^2 - z_2^2|^2 \right) + 2|z_2|^2$

$= 2(|z_1|^2 + |z_2|^2) + 2|z_1^2 - z_2^2|$

$= |z_1 + z_2|^2 + |z_1 - z_2|^2 + 2|z_1 + z_2||z_1 - z_2|$

$= \left(|z_1 + z_2| + |z_1 - z_2|\right)^2$

Therefore

$$\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right| = |z_1 + z_2$$

$| + |z_1 - z_2|$

Hence (B) is true. Also

$$\left|\frac{z_1 + z_2}{2} + \sqrt{z_1z_2}\right| + \left|\frac{z_1 + z_2}{2} - \sqrt{z_1z_2}\right|$$

$$= \frac{1}{2}\left|\sqrt{z_1} + \sqrt{z_2}\right|^2 + \frac{1}{2}\left|\sqrt{z_1} - \sqrt{z_2}\right|^2$$

$$= \frac{1}{2}\left[2\left|\sqrt{z_1}\right|^2 + 2\left|\sqrt{z_2}\right|^2\right] = |z_1| + |z_2|$$

Therefore (C) is true. **Ans.**

Q.10 (BCD)

$z_2 = i$; $z_3 = -1 + i$;

$z_4 = -i$; $z_5 = -1 + i$

$|z_{2050}| = 1$, $|z_{2017}| = \sqrt{2}$,

$|z_{2016}| = 1$, $|z_{2111}| = \sqrt{2}$

Q.11 (ABCD)

$$\frac{z^n - 1}{z - 1} = (z - \alpha_1)(z - \alpha_2)..........(z - \alpha_{n-1})$$

put $z = i$

$$\prod_{r=1}^{n-1}(i - \alpha_r) = \frac{i^n - 1}{i - 1} = \begin{cases} 0 & \text{if } n = 4k \\ 1 & \text{if } n = 4k+1 \\ 1+i & \text{if } n = 4k+2 \\ i & \text{if } n = 4k+3 \end{cases}$$

Q.12 (ABC)

$(z-1)(z-\alpha_1)(z-\alpha_2)\ ....\ (z-\alpha_{n-1}) = z^n - 1$ ..(1)

$$\underset{z\to1}{\text{Limit}}\ (z-\alpha_1)(z-\alpha_2)\ .....\ (z-\alpha_{n-1}) = \underset{z\to1}{\text{Limit}}\ \frac{z^n-1}{z-1} = n$$

hence $(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)........(1-\alpha_{n-1}) = n$.

put z = 2 in equation (1) we get $(2-\alpha_1)(2-\alpha_2)(2-\alpha_3)........(2-\alpha_{n-1}) = 2^n - 1$

$(z-1)(z-\alpha_1)(z-\alpha_2)\ ....\ (z-\alpha_{n-1}) = z^n - 1$

$(z-\alpha_1)(z-\alpha_2)\ ....\ (z-\alpha_{n-1}) = 1 + z + z^2 + ... + z^{n-1}$

take log on both sides we get

$\log(z-\alpha_1) + \log(z-\alpha_2) + ... + \log(z-\alpha_n) = \log(1 + z + ... + z^{n-1})$

differentiate and put z = 1, we get

$$\frac{1}{1-\alpha_1}+\frac{1}{1-\alpha_2}+......+\frac{1}{1-\alpha_n} = \frac{n-1}{2}$$

Q.13 (ABC)

Let $S = (n-1)\alpha + (n-2)\alpha^2 + ...+\alpha^{n-1}$

$\alpha S =$` $+ (n-1)\alpha^2 + ...$ $+ 2\alpha^{n-1} + \alpha^n$

$(1-\alpha) S = (n-1)\alpha - \alpha^2 - \alpha^3 - .... - \alpha^{n-1} - \alpha^n$

(as $\alpha^n = 1$)

$= n\alpha - (1 + \alpha + ... + \alpha^{n-1})$

$$S = \frac{n\alpha}{1-\alpha}$$

If $\alpha = e^{\frac{2\pi}{n}i}$ and comparing imaginary parts

$$\sum_{r=1}^{n-1}(n-r)\sin\left(\frac{2r\pi}{n}\right) = \text{Img}\left(\frac{n(\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n})}{1-\cos\frac{2\pi}{n}-i\sin\frac{2\pi}{n}}\right)$$

$$= \text{Img}\left(\frac{ne^{i\frac{2\pi}{n}}}{\left(1-e^{i\frac{2\pi}{n}}\right)}\right) = \text{Img}\left(\frac{n\,e^{i\frac{\pi}{n}}}{-2i\sin\left(\frac{\pi}{n}\right)}\right)$$

$$= I_m\left(\frac{n\,i\,e^{\frac{\pi}{n}i}}{2\sin\frac{\pi}{n}}\right) = \frac{n}{2}\cot\frac{\pi}{n}$$

Q.14 (BC)

$z = re^{i\theta}$

$r^2e^{i\theta 2} + r^2e^{i\theta} + r^2 = 0$

$r^2\,[e^{i2\theta} + e^{i\theta} + 1] = 0$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$z = k\omega$ or $k\omega^2$ where $k > 0$

Q.15 (ABCD)

$z^{10} - z^5 - 992 = 0$

$z^5 = 32$ or $z^5 = -31$

$$z = 2.\left(\cos\frac{2r\pi}{5}+i\sin\frac{2r\pi}{5}\right)$$

r = 0, 1, 2, 3, 4

for n = 2,3 roots have negative real part.

$z^5 = -31$

$$z = (31)^{\frac{1}{5}}\left[\cos(2r+1)\frac{\pi}{5}+i\sin(2r+1)\frac{\pi}{5}\right]$$

r = 1, 2, 3

⇒ roots have negative real part

5 roots have negative real part.

Q.16 (ABCD)

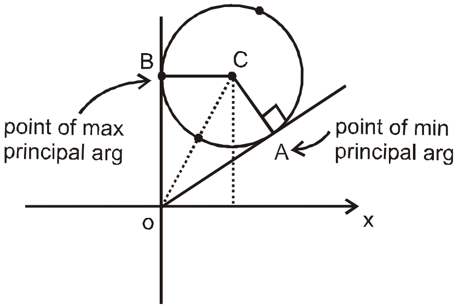
max | z | = d + r = $\sqrt{5}$ + 1

min | z | = d – r = $\sqrt{5}$ – 1

d = OC = $\sqrt{5}$

r = 1

$$\theta = \angle OCX = \tan^{-1}\frac{2}{1}$$



$$\alpha = \angle OCA = \tan^{-1}\frac{1}{2} \qquad \left(\because\ \sin\alpha = \frac{1}{\sqrt{5}}\right)$$

So principal Arg of A = $\theta - \alpha = \tan^{-1} 2 - \tan^{-1}\frac{1}{2}$

$$= \tan^{-1}\frac{2-\frac{1}{2}}{1+1} = \tan^{-1}\frac{3}{4}$$

Q.17 (ACD)

$z^3 + iz^2 + 2i = 0$, z = *i* satisfy

∴ z = *i* is a root

$(z-i)(z^2 + 2iz - 2) = 0$

⇒ $z = 1 - i, -1 - i, i$

vertices

A (0, 1), B (1, – 1), C (–1, –1)

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{vmatrix} \Rightarrow 2$$

AB = $\sqrt{5}$, BC = 2, AC = $\sqrt{5}$

$\therefore$ ΔABC is isoceles

$$r = \frac{\Delta}{s}$$

$$s = \frac{\sqrt{5}+\sqrt{5}+2}{2} = \sqrt{5} + 1$$

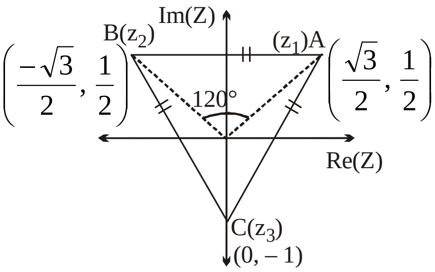
$$r = \frac{2}{\sqrt{5}+1} = \frac{2}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{\sqrt{5}-1}{2}$$

Q.18 (AB)

We have $z^4 = iz$

$\Rightarrow$ $z^3 = i$

$\Rightarrow$ $z = e^{i(4k+1)\frac{\pi}{6}}$ (Using D.M.T.)



Put k = 0, 1, 2, we get

$z_1 = e^{i\frac{\pi}{6}}$, $z_2 = e^{i\frac{5\pi}{6}}$ and $z_3 = e^{i\frac{3\pi}{2}}$

Clearly triangle formed by z_1, z_2 and z_3 is equilateral.

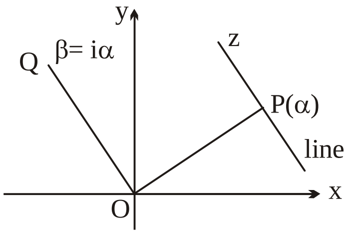
$\therefore$ centroid of ΔABC is (0, 0) and Area (ΔABC) = $\frac{3\sqrt{3}}{4}$

Q.19 (BD)

Required line is passing through P(α) and parallel to the vector $\overrightarrow{OQ}$

hence $z = \alpha + i\lambda\alpha,\ \lambda \in R$

$$\frac{z-\alpha}{\alpha} = \text{purely imaginary}$$



$\Rightarrow$ $\text{Re}\left(\frac{z-\alpha}{\alpha}\right) = 0$ $\Rightarrow$ **(B)**

(multiply N^r and D^r by $\overline{\alpha}$)

$\Rightarrow$ $\text{Re}\left((z-\alpha)\overline{\alpha}\right) = 0$ $\Rightarrow$ $\text{Re}\left(z\overline{\alpha} - |\,\overline{\alpha}\,|\right) = 0$

also

$$\frac{z-\alpha}{\alpha} + \frac{\overline{z}-\overline{\alpha}}{\overline{\alpha}} = 0$$

$$\overline{\alpha}(z-\alpha) + \alpha(\overline{z}-\overline{\alpha}) = 0$$

$$\overline{\alpha}\,z + \alpha\overline{z} - 2\,|\,\alpha\,|^2 = 0 \qquad \Rightarrow$$

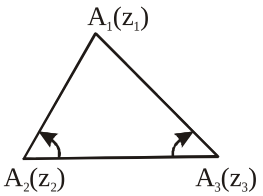
(D)

Q.20 (ABCD)

If $A_1A_2A_3$ is equilateral then rotate side A_2A_3 about (A_2) in anticlockwise sense

$$\frac{(z_1-z_2)}{|\,z_1-z_2\,|} = \frac{(z_3-z_2)}{|\,z_3-z_2\,|}e^{i\pi/3} \quad \ldots(i)$$

Rotate side A_1A_3 about (A_3) in anticlockwise sense then



$$\frac{z_2-z_3}{|\,z_2-z_3\,|} = \frac{z_1-z_3}{|\,z_1-z_3\,|}e^{i\pi/3} \quad \ldots(ii)$$

By (i) & (ii)

$$\frac{z_1-z_2}{z_3-z_2} = \frac{z_2-z_3}{z_1-z_3}$$

or $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

or $(z_1 + z_2\omega + z_3\omega^2)\ (z_1 + z_2\omega^2 + z_3\omega)$

for $\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_2 & z_3 & z_1 \end{vmatrix} = 0$

By expansion

$\Rightarrow$ $(z_2z_1 - z_3^2) - (z_1^2 - z_2z_3) + (z_1z_3 - z_2^2) = 0$

$\Rightarrow$ $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

Q.21 (AB)

Suppose that triangle ABC is equilateral.

Then $\frac{z_3 - z_1}{z_2 - z_1} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

and $\frac{z_1 - z_2}{z_3 - z_2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

Therefore

$(z_3 - z_1)(z_3 - z_2) = (z_2 - z_1)(z_1 - z_2)$

$z_3^2 - z_3z_2 - z_1z_3 + z_1z_2 = z_2z_1 - z_2^2 + z_1z_2$

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

Conversely, suppose that

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

Then,

$z_1(z_1 - z_2) + z_2(z_2 - z_3) + z_3(z_3 - z_1) = 0$

$(z_1 - z_2)^2 - (z_2 - z_3)(z_3 - z_1) = 0$

That is

$(z_1 - z_2)^2 = (z_2 - z_3)(z_3 - z_1)$

$(z_1 - z_2)^3 = (z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$

Similarly,

$(z_2 - z_2)^3 = (z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$

and $(z_3 - z_1)^3 = (z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$

Therefore

$(z_1 - z_2)^3 = (z_2 - z_3)^2 = (z_3 - z_1)^3$

$|z_1 - z_3| = |z_2 - z_3| = |z_3 - z_1|$

Therefore AB = BC = CA. That is ΔABC is equilateral.

We will prove that (B) is also correct. Suppose that ΔABC is equilateral. Then

$|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1| = k$ (say)

Let $\alpha = z_1 - z_2$, $\beta = z_2 - z_3$ and $\gamma = z_3 - z_1$. Then $\alpha + \beta + \gamma = 0$ and hence $\overline{\alpha} + \overline{\beta} + \overline{\gamma} = 0$. That is

$$\frac{k^2}{\alpha} + \frac{k^2}{\beta} + \frac{k^2}{\gamma} = 0 \text{ (since } \alpha\overline{\alpha} = |\alpha|^2 = k^2)$$

Therefore $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 0$

$$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$$

Conversely, suppose that $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 0$

Then $\frac{\alpha + \beta}{\alpha\beta} = -\frac{1}{\gamma}$

Therefore $-\gamma^2 = -\alpha\beta$

(since $\alpha + \beta = -\gamma$)

$\gamma^3 = \alpha\beta\gamma$

Similarly $\beta^3 = \alpha\beta\gamma = \alpha^3$

This gives $\alpha^3 = \beta^3 = \gamma^3$ and therefore $|\alpha| = |\beta| = |\gamma|$. That is

$|z_1 - z_2| = |z_3 - z_3| = |z_3 - z_1|$

Therefore ΔABC is equilateral.

Q.22 (AC)

AD is perpendicular to BC and therefore

$$\arg\left(\frac{z - z_1}{z_3 - z_2}\right) = \pm\frac{\pi}{2}$$

This implies that $(z - z_1)/(z_3 - z_2)$ is pure imaginary. Therefore

$$\frac{\overline{z} - \overline{z}_1}{\overline{z}_3 - \overline{z}_2} = -\left(\frac{z - z_1}{z_3 - z_2}\right)$$

$$\frac{\left(\frac{1}{z}\right) - \left(\frac{1}{z1}\right)}{\left(\frac{1}{z_3}\right) - \left(\frac{1}{z_2}\right)} = -\left(\frac{z - z_1}{z_3 - z_2}\right)$$

$$\left(\frac{z_1 - z}{z_2 - z_3}\right)\left(\frac{z_2z_3}{zz_1}\right) = -\left(\frac{z - z_1}{z_3 - z_2}\right)$$

$$\frac{z_2z_3}{zz_1} = -1 \text{ or } z = \frac{-z_2z_3}{z_1}$$

This implies (A) is correct.

Also, since the orthocentre H is $z_1 + z_2 + z_3$, we have

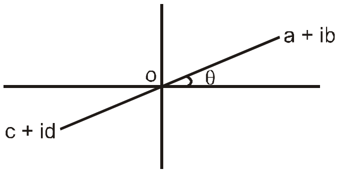
$BH = |z_1 + z_2 + z_3 - z_2| = |z_1 + z_3|$

and $BD = \left|z_2 + \frac{z_2z_3}{z_1}\right| = \frac{|z_2|}{|z_1|}|z_1 + z_3| = |z_1 + z_3|$

(since $|z_1| = 1 = |z_2|$)

Therefore, B is equidistant from H and D. similarly, C is equidistant from H and D. This gives that BC is the perpendicular bisector of HD and so H, D are reflections of each other through the side BC.

Q.23 (AB)



$|a + ib| = |(c + id)|$

$a + c = b + d$

Q.24 (ABC)

(i) $z_1\bar{z}_1 = 1$

$\Rightarrow \quad \bar{z}_1 = \dfrac{1}{z_1}$

(ii) $|z_1 + z_2....z_n| = |\bar{z}_1 + \bar{z}_2.....\bar{z}_n|$

$$= \left|\frac{1}{z_1} + \frac{1}{z_2} + ...\frac{1}{z_n}\right|$$

$z_1 + \dfrac{1}{z_1} = z_1 + \bar{z}_1 = 2\,\text{Re}(z_1)$ ∴ Centroid

$$= \frac{2[\text{Re}(z_1) + \text{Re}(z_2) + ... + \text{Re}(z_n)]}{n}$$

Whose imaginary part = 0

Q.25 (ABC)

$|z - z_0| = r$ represents a circles with centre z_0 and radius ‘r’

Squaring we get, $z\bar{z} - z_0\bar{z} - \bar{z}_0 z + z_0\bar{z}_0 - r^2 = 0$

or, $z\bar{z} + \alpha\bar{z} + \bar{\alpha}z + c = 0$ represents a circle whose centre is ‘$-\alpha$’ and

radius is $\sqrt{\alpha\bar{\alpha} - c}$.

$|z - \alpha|^2 = k^2\,|z - \beta|^2$

or $z\bar{z} - \alpha\bar{z} - \bar{\alpha}z + \alpha\bar{\alpha} = k^2\,(z\bar{z} - \beta\bar{z} - \bar{\beta}z + \beta\bar{\beta})$

or $z\bar{z}\,(1 - k^2) - \bar{z}\,(\alpha - k^2\beta) - z\,(\bar{\alpha} - k^2\bar{\beta}) + \alpha\bar{\alpha} - k^2\beta\bar{\beta} = 0$

Comparing with standard equation we get centre as $\dfrac{\alpha - k^2\beta}{1 - k^2}$ and radius is

$$\sqrt{\left(\frac{\alpha - k^2\beta}{1 - k^2}\right)\left(\frac{\bar{\alpha} - k^2\bar{\beta}}{1 - k^2}\right) - \frac{(\alpha\bar{\alpha} - k^2\beta\bar{\beta})}{1 - k^2}}$$

$$= \sqrt{\frac{\alpha\bar{\alpha} + k^4\beta\bar{\beta} - k^2\alpha\bar{\beta} - k^2\bar{\alpha}\beta - \alpha\bar{\alpha} + k^2\alpha\bar{\alpha} + k^2\beta\bar{\beta} - k^4\beta\bar{\beta}}{(1 - k^2)^2}}$$

$$= \sqrt{\frac{k^2(\alpha - \beta)(\bar{\alpha} - \bar{\beta})}{(1 - k^2)^2}} = \left|\frac{k}{1 - k^2}\right|\,|\alpha - \beta| = \left|\frac{k(\alpha - \beta)}{1 - k^2}\right|$$

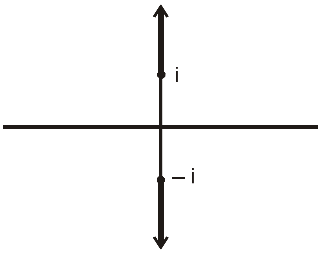
Q.26 (ACD)

$||z + i| - |z - i|| = k$

for $0 < k < 2$ its hyperbola having foci as i & $-i$.

for $k = 0$ $|z + i| = |z - i|$ which is perpendicular bisector of line joining $i, -i$

for $k = 2$ a pair of ray.



Q.27 (ABCD)

Putz = x + iy

$$\Rightarrow \frac{x^2}{\left(\frac{k^2 - 4}{4}\right)} + \frac{y^2}{\left(\frac{k^2}{4}\right)} = 1$$

if $k > 2$ $\Rightarrow$ ellipse

if $0 < k < 2$ $\Rightarrow$ hyperbola

if $k = 2$ $\Rightarrow x^2 = 0$

$\Rightarrow |y - 1| + |y + 1| = 2$

$\Rightarrow -1 \le y \le 1$

Hence z = x + iy lies on line segment joining – i and i

Q.28 (ACD)

All the three vertices lies on circle |z| = 1

take $z_1 = z_1,\ z_2 = z_1\omega,\ z_3 = z_1\omega^2$

so $z_1 + z_2 + z_3 = z_1(1 + \omega + \omega^2) = 0$

$z_1z_2z_3 = z_1^3$ ... (i)

$z_1z_2 + z_2z_3 + z_3z_1 = z_1^2\,(\omega + \omega^3 + \omega^2) = 0$

if $z_1 + z_2 + z_3 = 0$ then $z_1^3 + z_2^3 + z_3^3 = 3z_1z_2z_3$

from (i) $z_1^3 + z_2^3 + z_3^3 = 3z_1^3$

$z_2^3 + z_3^3 = 2z_1^3$

hence proved.

Q.29 (ABC)

$|z_1 - 1| = |z_2 - 1| = |z_3 - 1|$

1 is circumcentre of ΔABC

if $\dfrac{z_1 + z_2 + z_3}{3} = 1$

⇒ 1 is also centroid of ΔABC

so ΔABC is equilateral

$(z_1 + z_2 + z_3)^2 = z_1^2 + z_2^2 + z_3^2 + z_1z_2 + z_2z_3 + z_3z_1$

Q.30 (ABCD)

$$z = \frac{3}{2+\cos\theta+i\sin\theta} = \frac{3(2+\cos\theta-i\sin\theta)}{(2+\cos\theta)^2+\sin^2\theta}$$

$$= \frac{3.(2+\cos\theta-i\sin\theta)}{5+4\cos\theta}$$

For imaginary axis, real part = 0 i.e.

2 + cosθ = 0 which is not possible, so curve never meets the imaginary axis

For real axis Im z = 0

⇒ sin θ = 0 ⇒ θ = 0, π ∈ [0, 2π), so curve meets the real axis in two points.

$$|z| = 3.\frac{\sqrt{(2+\cos\theta)^2+(\sin\theta)^2}}{5+4\cos\theta} = 3(5 + 4\cos\theta)^{-1/2}$$

⇒ $|z|_{max} = 3$, $|z|_{min} = 1$

Q.31 (BD)

Let z = x + iy. Then $(x^2 + y^2) - 2i(x + iy) + 2c(1 + i) = 0$

Therefore

$x^2 + y^2 + 2y + i(2c - 2x) + 2c = 0$

$x^2 + y^2 + 2y + 2c = 0$ ....(1)

and $2c - 2x = 0$ or $x = c$ ....(2)

Substituting x = c in equation, we get that

$c^2 + y^2 + 2y + 2c = 0$ ....(3)

Equation (3) has solutions if $4 - 4(c^2 + 2c) \geq 0$, that is $1 - c^2 - 2c > 0$. Therefore

$(c + 1)^2 \leq 0$. Therefore

$(c + 1)^2 \leq 2$ or $-\sqrt{2} \leq c + 1 \leq \sqrt{2}$

$-\sqrt{2} - 1 \leq c \leq \sqrt{2} - 1$

It is given that $c \geq 0$. Therefore $0 \leq c \leq \sqrt{2} - 1$.

(i) If $c < \sqrt{2} - 1$, then z = c +

$\left(-1 \pm \sqrt{1 - 2c - c^2}\right) i$.

(ii) If c = $\sqrt{2}$ – 1, then $z = \left(\sqrt{2} - 1\right) - i$

(iii) If $c > \sqrt{2} - 1$, the equation has no solutions.

Q.32 (ABCD)

$\alpha + \beta = 3 + 4i$ $\gamma\delta = 13 + i$

$\gamma = \overline{\alpha}$ and $\delta = \overline{\beta}$

$\gamma + \delta = 3 - 4i$

$b = \Sigma\alpha\beta$

$= \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$= (a + b)(\delta + \gamma) + \alpha\beta + \gamma\delta = (3 + 4i)(3 - 4i) + 13 - i + 13 + i$

$= 9 + 16 + 26 = 51$

Q.33 (ABC)

If one root is i then other is – i

Let fourth root is α.

$2\alpha = \frac{3}{2}$ $\Rightarrow \alpha = \frac{3}{4}$

$\frac{-a}{2} = 2 + i + (-i) + \frac{3}{4} = \frac{11}{4}$

$a = \frac{-11}{2}$

Q.3 (ABCD)

(A) $x = e^{i\theta}$ $y = e^{i\phi}$

$x^n + \frac{1}{x^n} = 2\cos n\theta$

(B) $\frac{x}{y} + \frac{y}{x} = e^{i(\theta-\phi)} + e^{-i(\theta-\phi)} = 2\cos(\theta - \phi)$

(C) $xy + \frac{1}{xy} = 2\cos(\theta+\phi)$

Q.35 (ABCD)

Consider (A, B)

$(\cos x + i\sin x) + {}^nC_1(\cos 2x + i\sin 2x) + {}^nC_3(\cos 3x + i\sin 3x) + ... + {}^nC_n(\cos(n+1)x + i\sin(n+1)x)$

$= (\cos x + i\sin x)[1 + (\cos x + i\sin x)^nC_1 + (\cos x + i\sin x)^2\,{}^nC_2 + ... + {}^nC_n(\cos x + i\sin x)^n]$

$= (\cos x + i\sin x)[1 + (\cos x + i\sin x)]^n$

$= (\cos x + i\sin x)\ [1 + (\cos x + i\sin x)]^n$

$$= (\cos x + i\sin x)\left[2\cos\frac{x}{2}\left(\cos\frac{x}{2} + i\sin\frac{x}{2}\right)\right]^n$$

$$= (\cos x + i\sin x)\left[2^n\cos^n\frac{x}{2}\left(\cos\frac{nx}{2} + i\sin\frac{nx}{2}\right)\right]$$

$$= 2^n\cos^n\frac{x}{2}\left[\cos\left(\frac{nx}{2} + x\right) + i\sin\left(\frac{n+2}{2}\right)x\right]$$

Compare the root & imaginary parts we have (a) & (b) relation

Similarly C & D

Comprehension # 1 (Q. No. 36 to 38)

Q.36 (B)

Q.37 (C)

Q.38 (C)

Consider a complex number w = $\frac{z-i}{2z+1}$,

if w = $\frac{x+i(y-1)}{(2x+1)+2yi}$

w = $\frac{[x+i(y-1)][(2x+1)-2yi]}{\text{real quantity}}$

if w is purely imaginary

⇒ Re(w) = 0

i.e. $x(2x + 1) + 2y(y - 1) = 0$

⇒ $2x^2 + 2y^2 + x - 2y = 0$

$x^2 + y^2 + \frac{1}{2}x - y = 0$

circle with centre $\left(-\frac{1}{4}, \frac{1}{2}\right)$ and radius = $\sqrt{\frac{1}{16}+\frac{1}{4}}$

= $\frac{\sqrt{5}}{4}$ **Ans.**

if w is purely real ⇒ Im(w) = 0

$(y - 1)(2x + 1) - 2xy = 0$ ⇒ $y - 2x - 1 = 0$

⇒ $2x - y + 1 = 0$

If $| w | = 1$

⇒ $x^2 + (y - 1)^2 = (2x + 1)^2 + 4y^2$

$3(x^2 + y^2) + 4x + 2y = 0$

∴ Im (z) = 0

Comprehension # 2 (Q. No. 39 to 41)

Q.39 (D)

$2x(-\lambda) + (-6\lambda) + 7\lambda = 1$

⇒ $\lambda = -1$.

∴ a = 1, b = 6, c = – 7.

⇒ 7a + b + c = 6. **Ans.**

Q.40 (A)

$\omega = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

a = 2 $\lambda = -2$

b = 12

c = – 14 ⇒ $\frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{14}}$

⇒ $3\omega + 1 + 3\omega^2$

= – 3 + 1 = – 2.

Q.41 (B)

41. b = 6

$\lambda = -1$

a = 1

c = – 7

∴ $x^2 + 6x - 7 = 0$

⇒ roots = = – 7, 1

∴ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{6}{7}$

$$\sum_{h=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \frac{1}{1-\frac{6}{7}} = 7.$$

Comprehension # 3 (Q. No. 42 and 43)

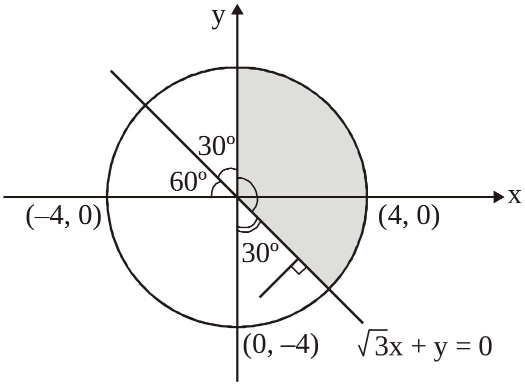
Let $S = S_1 \cap S_2 \cap S_3$, where $S_1 = \{z \in C : | z | < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : \text{Re } z > 0\}$.

Q.42 **[C]**

S_1 : $x^2 + y^2 < 16$

S_2 : $\text{Im}\left(\frac{(z-1+\sqrt{3}i)\,(1+\sqrt{3}i)}{4}\right) > 0$

or $\text{Im}\left(\frac{\{z-(1-\sqrt{3}i)\}\,\{1+\sqrt{3}i\}}{4}\right) > 0$



or $\text{Im}\left(\frac{(1+\sqrt{3}i)z-4}{4}\right) > 0$ ⇒

$\sqrt{3}\,x + y > 0$

S_3 : $x > 0$

$|1 - 3i - z| = |z - 1 + 3i| = |z - (1 - 3i)|$

Distance between [z and (1, –3)]

= $\left|\frac{\sqrt{3}-3}{\sqrt{3}+1}\right| = \frac{3-\sqrt{3}}{\sqrt{3}+1}$

Q.43 **[B]**

Area of S = $\frac{1}{2}\times 16\times\frac{5\pi}{6} = \frac{20\pi}{3}$ sq. unit

Comprehension # 4 (Q. No. 44 to 46)

Q.44 (C)

Q.45 (C)

Q.46 (A)

$C_1 : z + \bar{z} = 2|z-1|$

$2x = 2|x - 1 + iy|$

$x^2 = (x-1)^2 + y^2$

$\Rightarrow y^2 = 2x - 1$

$\Rightarrow y^2 = 2\left(x - \frac{1}{2}\right)$

$C_2 : \arg\left(z - (-1-i)\right) = \alpha$

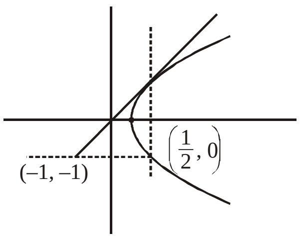
Curve C_2 is a ray emanating from (–1, –1) and making an angle α from the positive real axis

$\because$ C_1 and C_2 have exactly one common point

$\therefore$ C_2 must be a tangent to C_1 .

Solving, C_1 and C_2

$$y^2 = 2\left(\frac{y+1}{m} - 1\right) - 1$$



$my^2 = 2(y + 1 - m) - m$

$my^2 - 2y + 3m - 2 = 0$

$D = 0 \Rightarrow 4 - 4m(3m - 2) = 0$

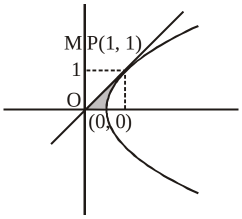
$3m^2 - 2m - 1 = 0 \Rightarrow (3m+1)(m-1) = 0 \Rightarrow m = \frac{-1}{3}, 1.$

$m = \frac{-1}{3}$ rejected $\therefore$ $m = 1$

Putting $y = x$ in the curve C_1

$x^2 = 2x - 1 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1 \Rightarrow P \equiv (1, 1)$

Complex number corresponding to P is $z_0 = 1 + i$



$|z_0| = \sqrt{2}$

Area of the shaded region

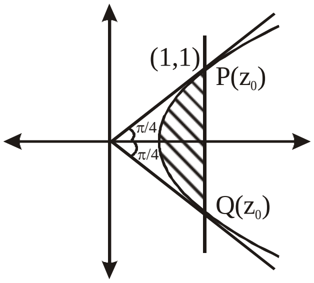
$$= \int_0^1 \frac{y^2+1}{2}\,dy - \text{Area of } \Delta OPM$$

$$= \frac{1}{2}\left(\frac{y^3}{3} + y\right)_0^1 - \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2}\cdot\frac{4}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq. units}$$

Area of the shaded region

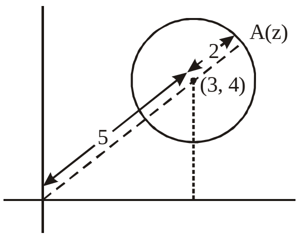
$$= 2\int_{1/2}^{1}\sqrt{2x-1}\,dx = \left(\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}\right)_{1/2}^{1}$$



$$= \frac{2}{3}[1 - 0] = \frac{2}{3} \text{ sq. units.}$$

Q.47 (A) S, (B) Q, (C) P, (D) R, S

(A) $|z - (3 + 4i)| = 2$ represents a circle whose centre is (3, 4) and radius is equal to 2.



From the figure it is clear that maximum value of $|z| = 7$.

(B) $|z - 12 - 6i| = |(z - i) + (12 - 5i)| \le |z - i| + |12 - 5i| < 1 + 13 = 14$

(C) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2) = 2(1 + 4) = 10$

(D) If $z = 1 + i$, then

$(z-1)^4 = i^4$

$z^4 - 4z^3 + 6z^2 - 4z + 1 = 1$

$(z^4 - 4z^3 + 7z^2 - 6z + 3) - z^2 + 2z - 2 = 1$

$z^4 - 4z^3 + 7z^2 - 6z + 3 = z^2 - 2z + 3 = (z-1)^2 + 2 = i^2 + 2 = 1$

$4(z^4 - 4z^3 + 7z^2 - 6z + 3) = 4$. **Ans.**

Q.48 **(A) → (q),**
(B) → (t), (C) → (p),(D) → (r)

(A) $\text{Re}\left(\frac{iz+1}{iz-1}\right) = 2$

$\Rightarrow \text{Re}\left(\frac{z-i}{z+1}\right) = 2$

Let $z = x + iy$ then

$\Rightarrow \quad \text{Re}\left(\frac{x+(y-1)i}{x+(y+1)i}\right) = 2$

$\Rightarrow \text{Re}\left(\frac{x^2+y^2-1+i\,2x}{x^2+(y+1)^2}\right) = 2$

$\Rightarrow \quad x^2 + y^2 - 1 = 2x^2 + 2(y+1)^2$

$\Rightarrow x^2 + y^2 + 4y + 3 = 0$

(B) It is a circle with diameter whose end points are $z_1(6, 1)$ and $z_2(4, -3)$ then equation of circle is

$\Rightarrow \quad (x-6)(x-4) + (y-1)(y+3) = 0$

$\Rightarrow \; x^2 + y^2 - 10x + 2y + 21 = 0$

$\Rightarrow \quad (x-5)^2 + (y+1)^2 = 5$

(C) Let $z = x + iy$

$\text{Im}\left(\frac{2z+1}{1+iz}\right) = 2$

$\Rightarrow \quad \left\{\left(\frac{(2x+1)+2i\,y}{(1-y)^2+x^2}\right)((1-y)-i\,x)\right\}$

$\Rightarrow \quad -x(2x+1) + 2y(1-y) = 2(1-y)^2 + 2x^2$

$\Rightarrow \quad 4x^2 + 4y^2 + x - 6y + 2 = 0$

(D) $\left|\frac{2z-i}{z+1}\right| = 1$

Let $z = x + iy$

$\Rightarrow \quad |2z - i|^2 = |z+1|^2$

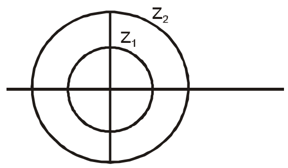
$\Rightarrow \quad |2x + 2iy - 1|^2 = |(x+1) + iy|^2$

$\Rightarrow \quad (2x)^2 + (2y-1)^2 = (x+1)^2 + y^2$

$\Rightarrow \; 3(x^2 + y^2) - 2x - 4y = 0$

Q.49 (A) → (p); (B) → (q); (C) → (r); (D) → (s)

(A) $|z_1 + z_2| \le |z_1| + |z_2| \le 2 + 1 \le 3$



(A) → P

(B) $|z_1 - z_2|$

$\Rightarrow$ minimum distance b/w z_1 & z_2 = 1

B → q

(C) $|2z_1 + 3z_2|$ minimum is $= 6 - 2 = 4$

(C) → r

(D) $|z_1 - 2z_2| \le |z_1| + |-2z_2|$

$1 + 4 \le 5$

(D) → s

Q.50 (A) → (s); (B) → (r); (C) → (q); (D) → (p)

(A) Z

$$= \frac{(1+i)^5(1+\sqrt{3}i)^2}{-2i(-\sqrt{3}+i)} = \frac{\left(\sqrt{2}^5\right)\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^5 2^2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2}{2i2\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)}$$

$\Rightarrow \quad \text{Argument} = \frac{5\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{2} + \frac{\pi}{6} = \frac{19\pi}{12}$

Therefore, the principal argument is $-5\pi/12$

(B) $\sin\frac{6\pi}{5} + i\left(1+\cos\frac{6\pi}{5}\right)$

lies in 2nd quadrant and

$$\left|\frac{1+\cos\frac{6\pi}{5}}{\sin\frac{6\pi}{5}}\right| = \left|\cot\left(\frac{3\pi}{5}\right)\right| = \left|\cot\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right| = \tan\frac{\pi}{10}$$

2nd quadrant $\quad \Rightarrow \quad \pi - \frac{\pi}{10}$

(C) $z = 1 + \cos\frac{11\pi}{9} + i\sin\frac{11\pi}{9} = \left(-2\cos\frac{11\pi}{18}\right)$

$\left[\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}\right](-1)$

$|z| = -2\cos\frac{11\pi}{18} = 2\cos\frac{7\pi}{18}$

$\arg(z) = \frac{11\pi}{18} - \pi = \frac{-7\pi}{18}$

(D) $Z = \sin x \sin(x-60)\sin(x+60)$

Now $Z = -\frac{1}{4}\sin 3x \qquad 3x \in (0, \pi)$

Z is a negative real number

hence principal arguments is π

Q.51 A → s; B → r; C → p; D → q.

A. $z^4 - 1 = 0 \Rightarrow z^4 = 1 = \cos 0 + i \sin 0$

$\Rightarrow z = (\cos 0 + i \sin 0)^{1/4} = \cos 0 + i \sin 0$

B. $z^4 + 1 = 0 \Rightarrow z^4 = -1 = \cos \pi + i \sin \pi$

$\Rightarrow z = (\cos \pi + i \sin \pi)^{1/4} = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$

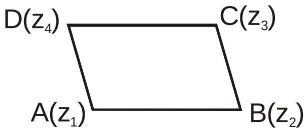
C. $iz^4 + 1 = 0 \Rightarrow z^4 = i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

$\Rightarrow z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/4} = \cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$

D. $iz^4 - 1 = 0 \Rightarrow z^4 = -i = \cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$

$\Rightarrow z = \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^{1/4} = \cos \frac{\pi}{8} - i \sin \frac{\pi}{8}$

Q.52 a → p, r; b → p,q,r, t; c → p,r,s; d → p,q,r,s, t.



in parallelogram, the mid-points of diagonals coincide $\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$

$\Rightarrow z_1 - z_4 = z_2 - z_3$

also in parallelogram, AB||CD.

Hence $\arg\left(\frac{z_1 - z_2}{z_3 - z_4}\right) = 0$

$\Rightarrow \frac{z_1 - z_2}{z_3 - z_4}$ is purely real

in rectangle, adjacent sided are perpendicular.

Hence $\arg\left(\frac{z_1 - z_2}{z_3 - z_2}\right) = \frac{\pi}{2} \Rightarrow \frac{z_1 - z_2}{z_3 - z_2}$ is purely imaginary

also is rectangle, AC = BD $\Rightarrow |z_1 - z_2| = |z_2 - z_4|$

in rhombus, AC ⊥ BD

$\Rightarrow \frac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary.

Q.53 (A) → (p); (B) → (s); (C) → (r); (D) → (s)

Given identity

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0$

$= \sin\alpha + \sin\beta + \sin\gamma$

Let $a = \cos\alpha + i \sin\alpha$, $b = \cos\beta + i \sin\beta$,

$c = \cos\gamma + i \sin\gamma$

$\Rightarrow a + b + c = 0$

$\Rightarrow \bar{a} + \bar{b} + \bar{c} = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \; (\because a\bar{a} = 1)$

$\Rightarrow ab + bc + ca = 0$

$\Rightarrow e^{i(\alpha + \beta)} + e^{i(\beta + \gamma)} + e^{i(\gamma + \alpha)} = 0$

$\Rightarrow \sum \cos(\alpha + \beta) = 0$ and $\sum \sin(\alpha + \beta) = 0$

$\Rightarrow$ (A) → P

$\because a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

equating real and imaginary parts,

$\sum \cos 3\alpha = 3\cos(\alpha + \beta + \gamma)$ and

$\Rightarrow$ (C) → (r)

(q) $\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{3abc} = \frac{3abc}{abc} = 3$

$\therefore \Sigma\cos(2\alpha - \beta - \gamma) = 3 \quad \because a + b + c = 0$

$e^{i\alpha} + e^{i\beta} + e^{i\gamma} = 0$

$\Rightarrow e^{i(\alpha + \theta)} + e^{i(\beta + \theta)} + e^{i(\gamma + \theta)} = 0$

$\Rightarrow \sum \cos(\alpha + \theta) = 0$

$\Rightarrow \sum \cos^3(\alpha + \theta) = 3\,\Pi\cos(\alpha + \theta)$

$\Rightarrow$ (D) → (s)

NUMERICAL VALUE BASED

Q.1 (99)

$N = (a + ib)^3 - 107i$

$= (a^3 - 3ab^2) + i[3a^2b - b^3] - 107i$ = Positive integer

$\therefore \quad 3a^2b - b^3 - 107 = 0$

$b(3a^2 - b^2) = 107$

$b = 1 \quad 3a^2 - b^2 = 107 \quad$ 107 is prime

$\Rightarrow \quad a = 6 \quad$ or $\quad b = 107$

$3a^2 - (107)^2 = 1$

a is not integer not possible

$\therefore \quad a = 6 \quad b = 1$

$N = 216 - 3 \times 6 = 216 - 18 = 198$.

Q.2 (32)

$z = \alpha \qquad \alpha \in R$

$\alpha^3 - (3 + i)\alpha + m + 2i = 0$

$\alpha^3 - 3\alpha + m = 0 \quad \& \quad -\alpha + 2 = 0$

$\alpha = 2$

$8 - 6 + m = 0 \quad \Rightarrow \quad m = -2$

$\Rightarrow \quad \alpha^4 + m^4 = 32$

Q.3 (25)

$z_1(z_1^2 - 3z_2^2) = 2$

...(1)

$z_2(3z_1^2 - z_2^2) = 11$ ...(2)

Now (1) + i(2)

$(z_1 + iz_2)^3 = 2 + 11i$ ...(3)

(i) – i(2)

$(z_1 - iz_2)^3 = 2 - 11i$ ...(4)

Multiply (3) & (4) we get

$(z_1^2 + z_2^2)^3 = 125$

$z_1^2 + z_2^2 = 5 = \lambda$

Q.4 (2)

lies $\alpha = a + ib$ $\gamma = c + id$

$\sqrt{a^2+b^2} + \sqrt{c^2+d^2} = p$

$f(z) = (4 + i)z^2 + z(a + ib) + (c + id)$

$f(1) = (4 + a + c) + i(1 + b + d)$

hence $1 + b + d = 0$ ...(1)

$f(i) = -(4 + i) + i(a + ib) + (c + id)$

$-4 - b + c + i(a + d - 1)$

hence $a + d = 1$ ...(2)

from (1) & (2) $a - b = 2$

hence there is no restriction of 'c' let c = 0

$|\alpha| + |\gamma| = \sqrt{a^2+b^2} + \sqrt{d^2} = \sqrt{4+2ab} + |d| \le$

$\sqrt{4+2ab} \ge \sqrt{2}$

with equality

$d = 0, a = 1, b = -1$

hence $p = \sqrt{2}$

Q.5 (15)

If $x = \alpha + i\beta$ is a root then

$$\frac{A_1^2}{\alpha - a_1 + i\beta} + \frac{A_2^2}{\alpha - a_2 + i\beta} + \ldots\ldots + \frac{A_n^2}{\alpha - a_n + i\beta} = K$$

& taking conjugate

$$\frac{A_1^2}{\alpha - a_1 - i\beta} + \frac{A_2^2}{\alpha - a_2 - i\beta} + \ldots\ldots + \frac{A_n^2}{\alpha - a_n - i\beta} = K$$

Substracting

$$\frac{2\beta A_1^2}{(\alpha - a_1)^2 + \beta^2} + \frac{2\beta A_2^2}{(\alpha - a_2)^2 + \beta^2} + \ldots\ldots + \frac{2\beta A_n^2}{(\alpha - a_n)^2 + \beta^2} = 0$$

$\Rightarrow \beta = 0$ $\Rightarrow x = \alpha + i\,0$

which is purely real. Hence true.

Q.6 (2)

Given

$2|z - 1| = z + \bar{z} \Rightarrow (x - 1)^2 + y^2 = x^2$

$y^2 = 2x - 1$ ..... (i)

Also Arg $(z_1 - z_2) = \frac{\pi}{4}$

both point lies on a straight line

y = x + c (say)

So curve .... (i)

$\Rightarrow y^2 = 2(y - c) - 1$

$\Rightarrow y^2 - 2y + 2c + 1 = 0$

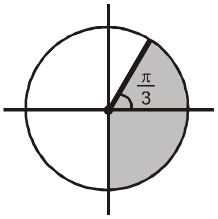
$\Rightarrow y_1 + y_2 = 2$

$\Rightarrow y^2 = 2(y - c) - 1$

$\Rightarrow y^2 - 2y + 2c + 1 = 0$

$\Rightarrow y_1 + y_2 = 2$

Q.7 (60)

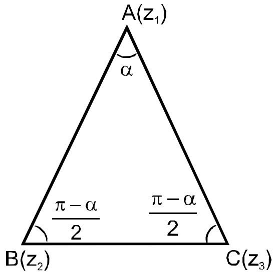


Area = $\frac{20\pi}{3}$

Q.8 (4)

$$\frac{z_1 - z_2}{z_3 - z_2} = \frac{AB}{BC}\, e^{i\left(\frac{\pi-\alpha}{2}\right)}$$

$$\frac{z_2 - z_3}{(z_1 - z_3)} = \frac{BC}{AC}\, e^{i\left(\frac{\pi-\alpha}{2}\right)}$$



$$\Rightarrow \frac{(z_1 - z_2)(-z_1 + z_3)}{(z_3 - z_2)^2} = \left(\frac{AB}{BC}\right)^2 .1 \;[\because AB = AC]$$

$$\Rightarrow (z_2 - z_3)^2 = (z_3 - z_1)(z_1 - z_2) \,.\, \left(\frac{BC}{AB}\right)^2$$

$$\left[\therefore \frac{BC/2}{AB} = \cos\left(\frac{\pi-\alpha}{2}\right)\right]$$

$= (z_3 - z_1)(z_1 - z_2)\left[2\cos\left(\frac{\pi-\alpha}{2}\right)\right]^2 = 4(z_3 - z_1)$

$(z_1 - z_2)\sin^2 \alpha/2$

Q.9 (1)

$$\arg\left(\frac{1 + Z_1 + Z_2 + \ldots\ldots\ldots + Z_7}{1 + Z_8 + Z_9 + Z_{10} + \ldots\ldots. + Z_{14}}\right)$$

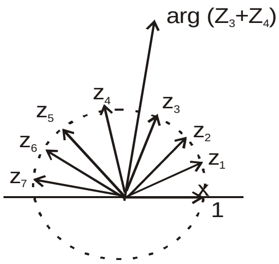
$$= \arg\left(\frac{1 + Z_1 + Z_2 + \ldots\ldots\ldots + Z_7}{1 + \overline{Z_1} + \overline{Z_2} + \ldots\ldots\ldots. + \overline{Z_7}}\right)$$

$= 2\arg(1 + Z_1 + Z_2 + \ldots\ldots\ldots + Z_7)$

$\left(\because \arg \frac{Z}{\bar{Z}} = 2\arg(Z)\right)$

$= (1 + \alpha + \alpha^2 + \ldots\ldots + \alpha^7)$ where $\alpha = e^{i\frac{2\pi}{15}}$

$$= \left(\frac{1-\alpha^8}{1-\alpha}\right)$$



$$= \left(\frac{1-\cos\frac{16\pi}{15}-i\sin\frac{16\pi}{15}}{1-\cos\frac{2\pi}{15}-i\sin\frac{2\pi}{15}}\right) =$$

$$\left(\frac{-2i^2\sin^2\frac{8\pi}{15}-2i\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}}{-2i^2\sin^2\frac{\pi}{15}-2i\sin\frac{\pi}{15}\cos\frac{\pi}{15}}\right)$$

$$= \left(\frac{\sin\frac{8\pi}{15}}{\sin\frac{\pi}{15}}\right)e^{i\frac{7\pi}{15}}$$

Hence 2 arg $(1+Z_1 + Z_2 + ...+Z_7) = \frac{14\pi}{15}$

Q.10 (13)

$$x = 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+....} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{1}{2}} = 3$$

$$y = 4^{\frac{1}{3}-\frac{1}{9}+\frac{1}{27}+...} = 4^{\frac{\frac{1}{3}}{1+\frac{1}{3}}} = 4^{\frac{1}{4}} = \sqrt{2}$$

$$z = \sum_{r=1}^{\infty}(1+i)^{-r} = \frac{1}{1+i}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^2}+...$$

$$= \frac{\frac{1}{1+i}}{1-\frac{1}{1+i}} = \frac{1}{i} = -i$$

Let $P = x + yz = 3 - i\sqrt{2}$ (fourth quadrant). Then , arg

$$P = -\tan^{-1}\left(\frac{\sqrt{2}}{3}\right) = -\tan^{-1}\left(\frac{\sqrt{a}}{b}\right)$$

Hence : a = 2, b = 3

Q.11 (25)

$z_1(z_1^2 - 3z_2^2) = 2$ ...(1)

$z_2(3z_1^2 - z_2^2) = 11$ ...(2)

Now (1) + i(2)

$(z_1 + iz_2)^3 = 2 + 11i$ ...(3)

(i) – i(2)

$(z_1 - iz_2)^3 = 2 - 11i$ ...(4)

Multiply (3) & (4) we get

$(z_1^2 + z_2^2)^3 = 125$

$z_1^2 + z_2^2 = 5 = \lambda$

KVPY
PREVIOUS YEAR'S

Q.1 (C)

$S = i + 2i^2 + 3i^3 + ... ni^n$

$$\frac{iS = i^2 + 2i^3 + ...(n-1)i^n + ni^{n+1}}{S(1-i) = i + i^2 + i^3 + ... + i^n ni^{n+1}}$$

$$S(1-i) = \frac{i(1-i^n)}{1-i} - ni^{n+1} \Rightarrow S = \underbrace{\frac{1-i^n}{-2i}}_{z_1\text{ (say)}} - \underbrace{\frac{ni^{n+1}}{1-i}}_{z_2\text{(say)}}$$

$$|z_1| = \frac{1}{\sqrt{2}} \text{ or } 0, |z_2| = \frac{n}{\sqrt{2}} = \frac{n}{2}\sqrt{2}$$

$$\frac{n}{2} = 18 \Rightarrow n = 36$$

Q.2 (B)

$|a + bw + cw^2|$

$|a - c + (b - c)w|$,

for maximum value taking a = 1 , c = –1, b = 1

$|a + bw + cw^2| = |2 + 2w| = 2|w^2| = 2$

Q.3 (C)

$$\sum_{k=0}^{n} {}^nC_k\omega^k = {}^nC_0 + {}^nC_1\omega + ... + {}^nC_n\omega^n$$

$$= (1+\omega)^n = (-\omega^2)^n$$

$$= (-1)^n\,\omega^{2n}$$

$$\therefore \left|e^{(-1)n}\omega^{2n}\right| = \left|e(-\omega^2)^n\right|$$

$$= \left|e\left(-\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right)^n\right|$$

$$= \left|e^{\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}}\right|$$

$$= \left|e^{\cos\frac{n\pi}{3}}\right| \text{can have values}$$

$$= \left\{e^1, e^{1/2}, e^{-1/2,}e^{-1}\right\}$$

Four values.

Q.4 (A)

$|z^3 + z^{-3}| \le 2$

$$\left|z^3 + \frac{1}{z^3}\right| \le 2$$

$$\left|\left(z+\frac{1}{z}\right)\left(z^2+\frac{1}{z^2}-1\right)\right| \le 2$$

$$\left|\left(z+\frac{1}{z}\right)\left(\left(z+\frac{1}{z}\right)^2-3\right)\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left|\left(z+\frac{1}{z}\right)^2-3\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left\{\left|\left|z+\frac{1}{z}\right|^2-3\right|\right\} \le 2 \qquad \{\because\ |z_1-z_2| > ||z_1|-|z_2||\}$$

$t|t^2 - 3| \le 2$ $\qquad$ $(t \ge 0)$ where $t = \left|z+\frac{1}{z}\right|$

$t \ge \sqrt{3}$	$0 \le t < \sqrt{3}$
$t(t^2–3) \le 2$	$t(3 – t^2) \le 2$
$t^3 – 3t – 2 \le 0$	$3t – t^3 \le 2$
$(t–2)(t + 1)^2 \le 0$	$t^3 – 3t + 2 \ge 0$
	$(t – 1)^2 (t + 2) \ge 0$
$t – 2 \le 0$	$t \ge –2$
$t \le 2$	

$t \in [0, \sqrt{3})$

$$\left|z+\frac{1}{z}\right|_{\max} = 2$$

Q.5 (D)

$e^{i\pi rn}$ is always a finite set when r is a rational & is infinite when $r = \frac{1}{\pi}$.

Q.6 (C)

$$|a| < 1 \;\&\; w = \left(\frac{z-a}{1-\bar{a}z}\right) \Rightarrow w - \bar{a}\,zw = z - a$$

$$\Rightarrow w + a = z\left(1+\bar{a}\,w\right)$$

$$z = \frac{w+a}{1+\bar{a}w}$$

Given $|z| < 1$

$$\left|\frac{w+a}{1+\bar{a}w}\right| < 1 \Rightarrow |w+a|^2 < |1+\bar{a}w|^2$$

$$\Rightarrow (w+a)\left(\overline{w}+\bar{a}\right) < (1+\bar{a}w)\left(1+a\overline{w}\right)$$

$$\Rightarrow w\overline{w} + w\bar{a} + a\overline{w} + a\bar{a} < 1 + \bar{a}w + a\overline{w} + a\bar{a}w\overline{w}$$

$$\Rightarrow a\bar{a}.w\overline{w} - w\overline{w} - a\bar{a} + 1 > 0$$

$$\Rightarrow |a|^2|w|^2 - |w|^2|a|^2 + 1 > 0$$

$$\Rightarrow (|a|^2 - 1)(|w|^2 - 1) > 0$$

Given $|a| < 1$, $|w|^2 – 1 < 0$

$|w| < 1$ & $|z| < 1$

Q.7 (C)

$$\cos x = \frac{e^{ix}+e^{-ix}}{2}$$

$$\cos 60x = \frac{e^{60ix}+e^{-60ix}}{2}$$

$$\cos 60x = \frac{(\cos x + i\sin x)^{60} + (\cos x - i\sin x)^{60}}{2}$$

$$\frac{1}{2} = \frac{\cos^{60} + 1° + 60c_2 \cos^{58} i\sin^2 1 + ...\infty)^{60} c_{60} \sin^{60} 1°}{2}$$

Change all sin 1º to cos1º. using the identity $\sin^2 1° =$ 1 $\cos^2$ 1º equation with root cos 1° so it is algebraic. Similarly for b = sin 1 also algebraic.

let a poly. $– x^2 + 1 \cos^2 1º = 0$

$– x^2 + 1 = – \sin^2 1º = 0$

Q.8 (C)

Let $z = e^{i\alpha}$ and $w = e^{i\beta}$

$z^2 + w^2 = 1 \qquad \Rightarrow \; e^{i2\alpha} + e^{i2\beta} = 1$

$\Rightarrow$ cos 2α + cos 2β = 1 and sin 2α + sin 2β = 0

$\Rightarrow$ 2 cos (α + β) cos (α – β) = 1

and 2 sin (α + β) cos (α – β) = 0

$\Rightarrow$ sin (α + β) = 0 $\qquad \Rightarrow \qquad$ α + β = nπ

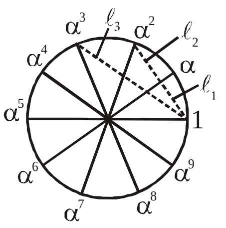
for α + β = 0, we have cos 2α = $\frac{1}{2}$

$\Rightarrow$ 4 pairs (α, β) for α ∈ [0, 2π)

for α + β = π, we have cos 2α = $\frac{1}{2}$

$\Rightarrow$ 4 pairs (α, β) for α ∈ [0, 2π)

Q.9 **(A)**



Let $\alpha = e\left(i\frac{2\pi}{10}\right) = e^{i\frac{\pi}{5}}$

Now, $z^{10} - 1 = (z - 1)(z - a)...(z - \alpha^9)$ ....(1)

so, $\ell_1 \ell_2 ... \ell_9 = |1 - a|\,|1 - \alpha^2|....|1 - \alpha^9|$

$= |(1 - \alpha)(1 - \alpha^2) ... (1 - \alpha^9)|$

$$= \left|\lim_{z\to 1}\frac{z^{10}-1}{z-1} = 10\right|$$

Q.10 **(A)**

$$10z\overline{z} - 3((z+\overline{z}^2) - 2z\overline{z}) + 4i((z+\overline{z})(z-\overline{z})) = 0$$

Let z = x + iy

$10(x^2 + y^2) - 3(4x^2 - 2x^2 - 2y^2) + 4i(2x(2iy)) = 0$

$\Rightarrow 4x^2 + 16y^2 - 16xy = 0$

$\Rightarrow x^2 - 4xy + 4y^2 = 0$

$\Rightarrow (x - 2y)^2 = 0 \Rightarrow x = 2y$

$\therefore$ straight line

JEE MAIN
PREVIOUS YEAR

Q.1 (4)

$L_1 = (2 - i)(x + iy) = 4i + (2 + i)(x - iy)$

$\Rightarrow (2x + y) + i(2y - x) = (4 + x - 2y)i + (2x + y)$

$\Rightarrow 2y - x = 4 + x - 2y$

$\Rightarrow 4y - 2x = 4$

$\Rightarrow x - 2y + 2 = 0$

$L_2 = (2 + i)(x + iy) = (i - 2)(x - iy)$

$\Rightarrow 2x - y + i(2y + x) = (-2x + y) + i(x + 2y)$

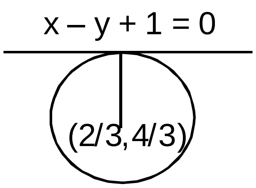
$\Rightarrow 2x - y = -2x + y$

$\Rightarrow 2x - y = 0 \qquad \Rightarrow x - 2(2x) + 2 = 0$

$\Rightarrow -3x + 2 = 0 \qquad \Rightarrow x = \frac{2}{3}, \; y = \frac{4}{3}$

$(i - 1)(x + iy) + (i + 1)(x - iy) + 2i = 0$

$-x - y + x + y = 0, \; x - y + x - y + 2 = 0$



$$r = \left|\frac{\frac{2}{3} - \frac{4}{3} + 1}{\sqrt{2}}\right| = \left|\frac{1}{3\sqrt{2}}\right|$$

Q.2 (3)

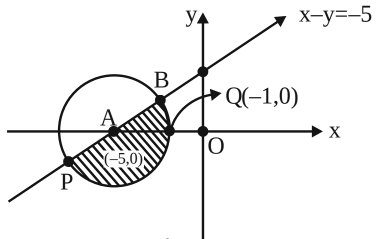
$n = 1, n = -\omega, n = -\omega^2$

$\alpha = 1, \beta = -\omega, \gamma = -\omega^2$

$E = 1 + \omega^{162} + (\omega^2)^{162}$

$= 3$

Q.3 (48)



Locus of z

$\therefore P \equiv \left(-5 - 2\sqrt{2}, -2\sqrt{2}\right)$

$\therefore (PQ)^2 |_{max} = 32 + 16\sqrt{2}$

$\alpha = 32$

$\beta = 16$

$\alpha + \beta = 48$

Q.4 (10)

$x + iy + \in \sqrt{(x-1)^2 + y^2} + 2i = 0$

$y + 2 = 0$ and $x + \sqrt{(x-1)^2 + y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$$\alpha^2 = \frac{x^2}{x^2 - 2x + 5}$$

$$\alpha^2 \in \left[0, \frac{5}{4}\right] \;\; \therefore \; \alpha^2 \in \left[0, \frac{5}{4}\right]$$

$$\therefore \; \alpha^2 \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$$

then $4[(\alpha_{max})^2 + (\alpha_{min})^2] = 4\left[\frac{5}{4} + \frac{5}{4}\right] = 10$

Q.5 **(2)**

$$\log g_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \le 2$$

$$\frac{|z|+11}{(|z|-1)^2} \ge \frac{1}{2}$$

$2|z| + 22 \ge (|z| - 1)^2$

$2|z| + 22 \ge |z|^2 + 1 - 2|z|$

$z|^2 - 4|z| - 21 \le 0$

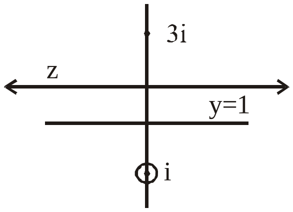
$\Rightarrow |z| \le 7$

$\therefore$ Largest value of $|z|$ is 7

Q.6 **(4)**

$\omega = z\overline{z} - 2z + 2$

$$\left|\frac{z+i}{z-3i}\right| = 1$$



$\Rightarrow |z + i| = |z - 3i|$

$\Rightarrow z = x + i, \; x \in \mathbb{R}$

$\omega = (x + i)(x - i) - 2(x + i) + 2$

$= x2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min (Re(ω)), x = 1

$\Rightarrow \omega = 2 ? 2i = 2(1 - i) = 2\sqrt{2}\,e^{-i\frac{\pi}{4}}$

$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

n = 4

Q.7 **(1)**

$$\exp\left(\frac{(|z|+3)\ |(z|-1)}{||z|+1|}\ell n\,2\right) \geq \log_{\sqrt{2}}|5\sqrt{7}+9i|$$

$$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$$

$$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$$

$$\Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$$

$\Rightarrow (|z| + 3)(|z| - 1) \geq 3(|z| + 1)$

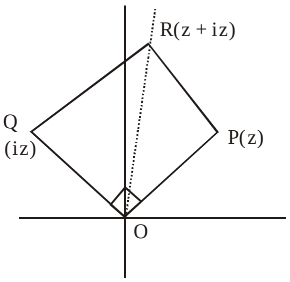
$|z|2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|2 + |z| - 6 \geq 0$

$\Rightarrow (|z| - 3)\ (|z| + 2) \geq 0 \Rightarrow |z| - 3 \geq 0$

$\Rightarrow |z| \geq 3 \Rightarrow |z|_{\min} = 3$

Q.8 **(2)**

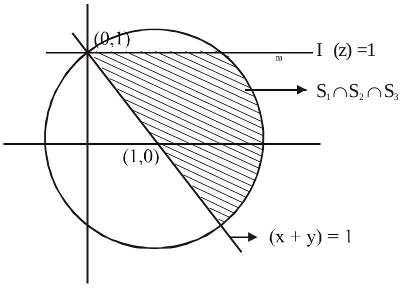


$$A = \frac{1}{2}|z|\ |iz|$$

$$= \frac{|z|^2}{2}$$

Q.9 **(3)**

For $|z-1| \leq \sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1, 0).



For S2

Let z = x + iy

Now, (1 – i) (z) = (1 – i) (x + iy)

Re((1 – i)z) = x + y

$\Rightarrow x + y \geq 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

Q.10 **(2)**

$azz + \alpha z + \alpha z + d = 0 \rightarrow$ Circle

centre $= \dfrac{-\alpha}{a}$ $\qquad 2 = \sqrt{\dfrac{\alpha\bar{\alpha}}{a^2} - \dfrac{d}{a}} = \sqrt{\dfrac{\alpha\bar{\alpha} - ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R\ .\ \{0\}$

Q.11 **(6)**

If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1{}^2 + z_2{}^2 = 0\ (z_1 + z_2) + z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow a^2 = 3 \times 12$

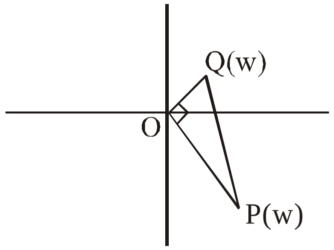
$\Rightarrow |a| = 6$

Q.12 **(2)**

$\mathbf{w = 1 -} \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \dfrac{1}{|w|} \Rightarrow |z| = \dfrac{1}{2}$

and $\text{amp}(z) = \dfrac{\pi}{2} + \text{amp}(w)$



$\Rightarrow$ Area of triangle $= \dfrac{1}{2}.OP.OQ$

$= \dfrac{1}{2}.2.\dfrac{1}{2} = \dfrac{1}{2}$

Q.13 **(2)**

Q.14 (1)

Q.15 (1)

Q.16 (4)

Q.17 (1)

Q.18 **(3)**

Q.19 **(1)**

Q.20 (5)

Q.21 **(4)**

Q.22 **(98)**

Q.23 (1)

Q.24 **(6)**

Q.25 (2)

Q.26 (13)

Q.27 (1)

Q.28 **(256)**

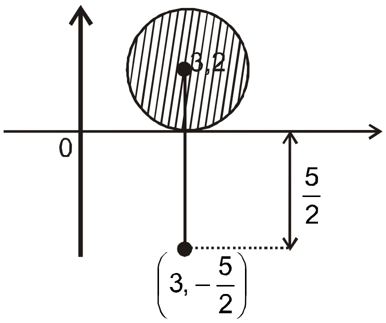
JEE-ADVANCED

PREVIOUS YEAR'S

Q.1 (5)

$$|2z - 6 + 5i| = 2\left|z - \left(3 - \frac{5i}{2}\right)\right|$$

for minimum $= 2 \times \frac{5}{2} = 5$



Q.2 3, Bonus (w = $e^{i\pi/3}$ is a typographical error, because of this the answer cannot be an integer.)

so lets assume ω = $e^{i2\pi/3}$, then the solution is following

$a + b + c = x$
$a + b\omega + c\omega^2 = y$
$a + b\omega^2 + c\omega = z$

$$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{x\bar{x} + y\bar{y} + z\bar{z}}{|a|^2 + |b|^2 + |c|^2}$$

$$= \frac{(a+b+c)(\bar{a}+\bar{b}+\bar{c}) + (a+b\omega+c\omega^2)(\bar{a}+\bar{b}\omega^2+\bar{c}\omega) + (a+b\omega^2+c\omega)(\bar{a}+\bar{b}\omega+\bar{c}\omega^2)}{|a|^2+|b|^2+|c|^2}$$

$$= \frac{3(|a|^2 + |b|^2 + |c|^2)}{|a|^2 + |b|^2 + |c|^2} = 3$$

Q.3 (D)

Here $z^2 + z + 1 - a = 0$

$$\Rightarrow \quad z = \frac{-1 \pm \sqrt{4a - 3}}{2}$$

Here $a \neq \frac{3}{4}$ otherwise z will be purely real.

Q.4 (C)

$|z - z_0| = r$
$|z - z_0| = 2r$
$|\alpha - z_0| = r$

$$\left|\frac{1}{\bar{\alpha}} - z_0\right| = 2r$$

$$\alpha\bar{\alpha} = |\alpha|^2$$

$$\left|\frac{\alpha}{|\alpha|^2} - z_0\right| = 2r$$

$$(\alpha - z_0)\left(\bar{\alpha} - \bar{z}_0\right) = r^2$$

$$\Rightarrow |\alpha|^2 - z_0\bar{\alpha} - \alpha\bar{z}_0 + |z_0|^2 = r^2$$

$$\left(\frac{\alpha}{|\alpha|^2} - z_0\right)\left(\frac{\bar{\alpha}}{|\alpha|^2} - \bar{z}_0\right) = 4r^2$$

$$\Rightarrow \frac{|\alpha|^2}{|\alpha|^4} - \frac{z_0\bar{\alpha}}{|\alpha|^2} - \frac{\bar{z}_0\alpha}{|\alpha|^2} + |z_0|^2 = 4r^2$$

$$1 - z_0\bar{\alpha} - \bar{z}_0\alpha + |z_0|^2\,|\alpha|^2 = 4r^2|\alpha|^2$$

$$\Rightarrow \quad (|\alpha^2| - 1) + |z_0|^2\,(1 - |\alpha|^2) = r^2\,(1 - 4\alpha^2)$$

$$(|\alpha|^2 - 1)\left(1 - \frac{r^2 + 2}{2}\right) = r^2(1 - 4|\alpha|^2)$$

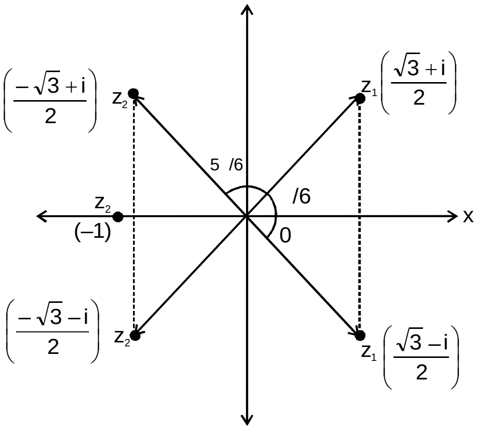
$$(|\alpha|^2 - 1)\left(\frac{-r^2}{2}\right) = r^2(1 - 4|\alpha|^2)$$

$|\alpha|^2 - 1 = -2 + 8|\alpha|^2$
$1 = 7|\alpha|^2$

$$\Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$$

Q.5 (C, D)

$$P = \omega^n = \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6},\ H_1 = \text{Re}z > 1/2$$



$$z_1 = P \cap H_1 = \frac{\sqrt{3}+i}{2},\ \frac{\sqrt{3}-i}{2}$$

$$z_2 = P \cap H_2 = -1,\ \frac{-\sqrt{3}+i}{2},\ \frac{-\sqrt{3}-i}{2}$$

$$\angle z_1 O z_2 = \frac{2\pi}{3}, \frac{5\pi}{6}, \pi$$

Q.6 (B, C, D)

n = 1

n = 2

$P=[\omega^2]$

$$P = \begin{bmatrix} \omega^2 & \omega^3 \\ \omega^3 & \omega^4 \end{bmatrix} = \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$$

$P^2 = [\omega^4] \neq 0$

$$P^2 = \begin{bmatrix} \omega^4 + 1 & .... \\ .... & .... \end{bmatrix} \neq 0$$

n = 3

$$P = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix}\begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Similarly $P^2 \neq 0$ when n is not multiple of 3.

Comprehension # 1 (Q. No. 7 to 8)

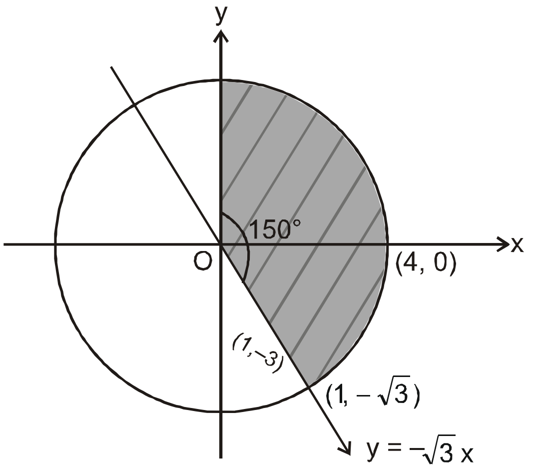
Q.7 (B)

Q.8 (C)

Sol.7 $S_1 : x^2 + y^2 < 16$

$$S_2 : \frac{z-1+\sqrt{3}i}{1-i\sqrt{3}} = \frac{(x-1)+i(y+\sqrt{3})}{1-\sqrt{3}\,i}$$

$$= \frac{\{(x-1)+i(y+\sqrt{3})\}\{1+\sqrt{3}\,i\}}{1+3}$$



$$S_2 : \frac{(x-1)\sqrt{3}+y+\sqrt{3}}{4} > 0$$

$S_2 : \sqrt{3}\,x + y > 0$ & $S_3 : x > 0$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 16 \times \frac{5\pi}{6}$$

$$= \frac{40\pi}{6} = \frac{20\pi}{3}$$

Sol.8 $\min\limits_{z \in S} |1 - 3i - z|$ = perpendicular length of point (1, –3) from line $\sqrt{3}\,x + y = 0$

$$\left|\frac{\sqrt{3}-3}{\sqrt{3+1}}\right| = \left|\frac{\sqrt{3}-3}{2}\right| = \frac{3-\sqrt{3}}{2}$$

Q.9 (C)

(P) $z_k z_j = 1 \quad \Rightarrow z_j = z_{10-k}$

Hence for each $k \in \{1, 2, 3, ... , 9\}$ there exists z_j such that $z_k \cdot z_j = 1$ True

(Q) $z_1 \cdot z = z_k$

$\Rightarrow z = z_{k-1}$ for k = 2, 3, 4, . . . , 9 &

z = 1 for k = 1

False

(R) $z_1, z_2, ... , z_9$ are roots of the equation $z^{10} = 1$ other then unity, hence

$$\frac{z^{10}-1}{z-1} = 1 + z + \ldots + z^9 = (z-z_1)(z-z_2) \ldots (z-z_9)$$

Substituting z = 1, we get

$$\frac{(1-z_1)(1-z_2)\ldots(1-z_9)}{10} = \frac{10}{10} = 1$$

(S) $1 - \sum\limits_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ = 1 – {sum of real parts of roots of $z^{10} = 1$ except 1}

= 1 – (–1) = 2

(as $1 + z_1 + z_2 + ... + z_9 = 0$)

$\Rightarrow \sum \mathrm{Re}(z_k) + 1 = 0$

Q.10 (4)

$$\alpha_k = \cos\frac{2k\pi}{14} + i\sin\frac{2k\pi}{14} = e^{i\frac{2k\pi}{14}}$$

$$\text{Now } \frac{\sum\limits_{k=1}^{12}\left|e^{\frac{i2(k+1)\pi}{14}} - e^{\frac{i2k\pi}{14}}\right|}{\sum\limits_{k=1}^{3}\left|e^{\frac{i(4k-1)\pi}{14}} - e^{\frac{i(4k-2)\pi}{14}}\right|} = \frac{\sum\limits_{k=1}^{12}\left|e^{\frac{i2\pi}{14}} - 1\right|}{\sum\limits_{k=1}^{3}\left|e^{\frac{i2\pi}{14}} - 1\right|}$$

$$= \frac{12}{3} = 4$$

Q.11 (1)

Let $z = \omega,\ z^2 = \omega^2$

$$\Rightarrow P = \begin{bmatrix} (-\omega)^r & (\omega^2)^s \\ (\omega^2)^s & (\omega)^r \end{bmatrix}$$

Now, $P^2 = -I$

$$\Rightarrow \left(-\omega\right)^{r}\left(\omega^{2}\right)^{s}+\omega^{2s+r}=0,\ \omega^{2r}+\left(\omega^{2}\right)^{2s}=-1$$

Only $r = s = 1$ satisfies this.
No. of ordered pairs $(r, s) = 1$

Q.12 (A, C, D)

$$z=x+iy=\frac{1}{a+ibt}$$

$$\Rightarrow x+iy=\frac{1}{a+ibt}\times\frac{a-ibt}{a-ibt}$$

$$\Rightarrow x=\frac{a}{a^{2}+b^{2}t^{2}},\ y=\frac{-bt}{a^{2}+b^{2}t^{2}}$$

So $t=-\dfrac{ay}{bx}$

$$\therefore x=\frac{a}{a^{2}+b^{2}\left(\dfrac{a^{2}y^{2}}{b^{2}x^{2}}\right)} \qquad \Rightarrow x^{2}+y^{2}-\frac{x}{a}=0$$

which represents a circle with radius $\dfrac{1}{2a}$ and center $\left(\dfrac{1}{2a},0\right)$ for a > 0

Q.13 **(B,D)**

$$\frac{a\left(x+iy\right)+b}{x+iy+1}=\frac{ax+b+iay}{x+1+iy}\times\frac{\left(x+1\right)-iy}{\left(x+1\right)-iy}=$$

$$\frac{\left(ax+b\right)\left(x+1\right)+ay^{2}}{\left(x+1\right)^{2}+y^{2}}+\frac{i\left(ay\left(x+1\right)-y\left(ax+b\right)\right)}{\left(x+1\right)^{2}+y^{2}}$$

$$\Rightarrow \frac{ay\left(x+1\right)-y\left(ax+b\right)}{\left(x+1\right)^{2}+y^{2}}=y$$

$$\Rightarrow \frac{ay-by}{\left(x+1\right)^{2}+y^{2}}=y \qquad (\because\ a-b=1\ ,\ y\neq 0)$$

$\Rightarrow (x + 1)^2 + y^2 = 1$

$\Rightarrow x + 1 = = \pm\sqrt{1-y^{2}} \qquad \Rightarrow \qquad x = -1\pm\sqrt{1=y^{2}}$

Q.14 **(A, B, D)**

(A) $\arg(-1 -i) = -\dfrac{3\pi}{4}$,

(B) $f(t) = \arg(-1 + it) = \begin{cases} \pi-\tan^{-1}(t), & t\geq 0 \\ -\pi+\tan^{-1}(t), & t<0 \end{cases}$

Discontinuous at t = 0.

(C) $\arg\left(\dfrac{z_1}{z_2}\right) - \arg(z_2) + 2n\pi - \arg(z_1) + \arg(z_2) = 2n\pi$.

(D) $\arg\left(\dfrac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right)=\pi$

$\Rightarrow \quad \dfrac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ is real.

$\Rightarrow \quad z, z_1, z_2, z_3$ are concyclic.

Q.15 **(A, C, D)**

Given

$sz\,|\,t\overline{z}+r=0$ ...(i)

on taking conjugate $\overline{s}\,\overline{z}+\overline{t}z+\overline{r}=0$ ...(ii)

from (1) and (2) elliminating $\overline{z}$

$$z\left(|\,s\,|^{2}-|\,t\,|^{2}\right)=\overline{r}t-r\overline{s}$$

(A) If $|s|\neq|t|$ then z has unique value

(B) If $|s|=|t|$ then $\overline{r}t-r\overline{s}$ may or may not be zero so L may be empty set

(C) locus of z is noll set or singleton set or a line in all cases it will intersect given circle at most two points.

(D) In this case locus of z is a line so L has infinite elements

Q.16 (B,C)

$$\left|z^{2}+z+1\right|=1$$

$$\Rightarrow \left|\left(z+\frac{1}{2}\right)^{2}+\frac{3}{4}\right|=1$$

$$\Rightarrow \left|\left(z+\frac{1}{2}\right)^{2}+\frac{3}{4}\right|\leq\left|z+\frac{1}{2}\right|^{2}+\frac{3}{4}$$

$$\Rightarrow 1\leq\left|z+\frac{1}{2}\right|^{2}+\frac{3}{4} \quad \Rightarrow \left|\left(z+\frac{1}{2}\right)\right|^{2}\geq\frac{1}{4}$$

$$\Rightarrow \left|z+\frac{1}{2}\right|\geq\frac{1}{2}$$

also $|(z^2+z)+1|=1\geq||z^2+z|-1|$
$\Rightarrow |z^2+z|-1\leq 1$
$\Rightarrow |z^2+z| \leq 2$
$\Rightarrow ||z^2|-|z|| \leq |z^2+z| \leq 2$
$\Rightarrow |r^2-r| \leq 2$
$\Rightarrow \qquad r = |z|\leq 2\ ;\ \forall z \in S$
Also we can always find root of the equation $z^2 + z + 1 = e^{i\theta}\ ;\ \forall\theta\in R$

Hence set 'S' is infinite

Q.17 (8)

Let z=x+iy

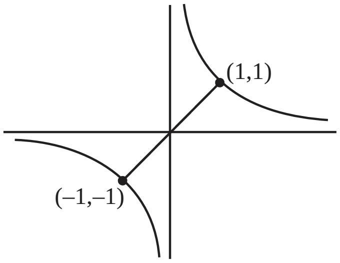
$z^4 - |z|^4 = 4iz^2$

$\Rightarrow z^4 - \left(z\overline{z}\right)^2 = 4iz^2$

$\Rightarrow z = 0$ or $z^2 - \left(\overline{z}\right)^2 = 4i$

$\Rightarrow 4ixy = 4i$

$\Rightarrow xy = 1$



$\left|z_1 - z_2\right|^2_{\min} = 8$

Q.18 (C)

$|z_1| = |z_2| = ... |z_{10}| = 1$

angle = $\frac{\text{arc}}{\text{rad}}$

$\theta_2 = \text{arc}(z_1z_2) > (z_2 > z_1)$

P : $|z_2 - z_1| + ... + |z_1 - z_{10}| \le \theta_1 + \theta_2 + ... + \theta_{10}$

$\Rightarrow |z_2 - z_1| + ... + |z_1 - z_{10}| \le 2\pi$ p is true

$z_1^2 = e^{i2\theta_1}, z_k^2 = z_{k-1}^2.e^{i2\theta_k}$

Let $2\theta_k = \alpha k$

$z_1^2 = e^{i\alpha_1}, z_k^2 = z_{k-1}^2.e^{i\alpha_k}$

$\alpha_1 + \alpha_2 + ... + \alpha_k = 4\pi$

one similar sense

$| z_1^{\ 2} - z_2^{\ 2} | + ... | z_1^{\ 2} - z_{10}^{\ 2} | \le 4\pi$

Q is also true

Q.19 (B,D)